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ABSTRACT

During the 1978-79 school year, 27,058 students were involved in the Minnesota Educational Assessment in Mathematics at three grade levels. Eighth graders were tested in the fall, fourth graders in the winter, and eleventh graders in the spring. The first chapter of this report presents a general overview of the assessment program. Chapter two provides an overview of student performance across grade levels, with trends identified as well as observations, conclusions, and recommendations. Chapters three, four, and five focus on data from the fourth, eighth, and eleventh grade assessments respectively. Appropriate comparisons are also provided with a similar assessment conducted during the 1974-75 school year. Performance of the Minnesota students compares favorably with performance on the National Assessment of Educational Progress, and this advantage has generally been maintained across the grade levels from 1974 to 1978. On overall performance, girls display a very slight advantage at grade four, which declines to a slight disadvantage at grade eleven. Generally, there has been a relative increase in the area of arithmetic skills, while there has been a decline in the area of problem solving. (MNS)

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ED248147

Mathmatics Performance

1978-1979 Minnesota Statewide Assessment Program

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MINNESOTA STATEWIDE EDUCATIONAL
ASSESSMENT
IN
MATHEMATICS
1978-79

FINAL REPORT

Minnesota Department of Education
Division of Special Services
Assessment Section

Division of Instruction
Elementary/Secondary Section

August, 1980

FOREWARD

Members of the Assessment Section and the Mathematics Unit of the Minnesota Department of Education would like to thank the many math educators and others who gave willingly of their time and efforts to make the mathematics assessment a successful endeavor. Subject specialists assisted in developing, field testing, critiquing, and finalizing items for the assessment. Mathematics educators also assisted in establishing criteria against which to judge student performance, and in completing this report.

The Assessment Section, by itself, cannot bring about the changes which data from this report suggest. Rather, it will take a widespread effort by individuals and groups to translate the data into improved programs for students in Minnesota schools.

Because this report represents the efforts of many Mathematics educators, it is hoped that the results will be utilized over the next several years to provide a focus for continued efforts to improve Mathematics education in Minnesota.

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CHAPTER 1

ASSESSMENT PROGRAM OVERVIEW

CHAPTER 1

ASSESSMENT PROGRAM OVERVIEW

INTRODUCTION

During the 1978-79 school year, 27,058 public school students were involved in the Minnesota Educational Assessment in Mathematics. Following a testing pattern established by the National Assessment of Educational Progress (NAEP), three grade levels were included in the assessment. Eighth graders were tested in the fall, fourth graders in the winter, and eleventh graders in the spring.

This chapter presents a general overview of the assessment program. Chapter two provides an overview of student performance across grade levels, with trends identified, as well as observations, conclusions, and recommendations. Chapters three, four, and five focus on data from the fourth, eighth, and eleventh grade assessments respectively. Since a similar assessment was conducted during the 1974-75 school year, appropriate comparisons are also provided.

PROGRAM PURPOSE AND ORIENTATION

The Minnesota Educational Assessment Program provides a means to collect statewide information over a broad range of curriculum areas at several grade levels. To ensure that the assessment reflects current educational goals and priorities, large numbers of Minnesota educators are involved in the various aspects of the assessment process. As a result, each assessment provides baseline data which can help facilitate and target reasoned instructional change.

Since the purpose of the assessment is curriculum analysis rather than the reporting of individual student performance, a random sample of students from across the state is selected.

When an assessment is planned, subject specialists and others are brought together to work with Department of Education personnel to identify the goals and objectives of teaching and learning in the subject area. Subject specialists are also integral in establishing criteria for performance as well as interpreting data, preparing reports, and formulating specific recommendations for improvement. As a result, each assessment provides an analysis of the strengths and weaknesses of public education in key areas of educational endeavor in Minnesota, and provides a focus for planned follow-up activities aimed at improving teaching and learning in the subject area.

INSTRUMENT DEVELOPMENT

The instruments in the Minnesota Educational Assessment Program are criterion-referenced. That is, questions are developed to measure student performance relative to specified goals and objectives. The primary source for item development are the Some Essential Learner Outcomes (SELO's) packages whose development has been coordinated within the Division of Instruction at the Minnesota State Department of Education. Results are essentially reported against these SELO's. The coordination of assessment procedures with Division of Instruction activities provides a continuity which enhances subject matter development. Specifically, the Minnesota Educational Assessment Program provides:

1. Statewide information on how Minnesota students perform in specific subject areas;
2. Information on student performance in categories of school districts;
3. Comparisons of Minnesota student performance with national groups of students;
4. An analysis of student performance, with corresponding observations, conclusions, and recommendations;

5. A reporting and dissemination plan aimed at improving student performance in areas where performance was determined to be low;

6. Materials and data for districts which choose the Local Assessment, or "Piggyback" Option, to analyze their curriculum; and,

7. Information for the schools sampled in the statewide assessment which can assist the education personnel in those schools to make instructional decisions.

The design of the Minnesota Educational Assessment Program does not provide for:

1. A comparison of individual school districts;

2. A teacher evaluation system; or,

3. A means to measure everything of importance relative to the education system.

The latter characteristics are beyond the scope and purposes for which the Minnesota Educational Assessment Program was designed.

While some questions are used at more than one grade level, for the most part questions are developed for students at a particular grade level. Therefore, while many of the same objectives are tested across grade levels, test questions reflect the maturity and knowledge level of students.

STUDENT SAMPLE SELECTION

There were a total of 27,058 students included in the mathematics assessment at grades four, eight, and eleven. For each grade level assessment, students were selected by means of a random sampling procedure from five categories of school districts. There were 8,337 fourth graders, or about 15% of the fourth grade public school population included in the assessment. A total of 8,921 eighth graders, or about 12% of the eighth grade public school population, and 9,800 eleventh graders, or about 13% of the eleventh grade public school population was included. Because of the size of the sample at each grade level and the random nature of the selection process, generalizations from the data to the total grade population may be made with a high degree of confidence.

ESTABLISHING CRITERIA FOR STUDENT PERFORMANCE

One of the important elements in the evaluation phase of the assessment program is the use of subject specialists to establish criteria for student performance. Typically, subject specialists establish a range of acceptable performance on items and item groupings prior to seeing student performance results. This same group then applies results against criteria in order to determine relative strengths and weaknesses in performance. In the mathematics assessment, this procedure was modified slightly.

Subject specialists did not establish a range for acceptable performance for items, since it was their opinion that desired scores would be very high on almost all items. Instead, it was decided that a single criterion, or point, would best serve as a benchmark against which to judge the adequacy of performance. Consequently, judgments regarding needs and strengths were, in this case, applied against a minimum acceptable level of performance. For the analysis of student performance, two separate groups were employed. One established criteria - the second provided need/strength ratings.

In establishing a level against which to judge student performance, subject specialists considered the difficulty of the item, the maturity level of students, the point at which the concept is generally introduced into the curriculum, and the level of emphasis it receives. It should be clear that judgments reflect not only ongoing programs to which students are exposed, but also reflect emerging curricular trends and subsequent skills necessary in that regard.

DISSEMINATION OF STATEWIDE ASSESSMENT RESULTS

The Minnesota Educational Assessment Program enables various audiences to have access to common data, which are based upon common outcomes, and which are gathered in a consistently valid manner. This common data base provides a mechanism for consistently monitoring student achievement and, in addition, it also provides the potential for improving teaching and learning within subject areas.

Completed assessment reports, which are presented to the Minnesota State Board of Education and appear in various media, serve as a basis for activities designed to impact upon schools and classrooms in Minnesota. These activities include working with Department of Education subject matter specialists and state organizations to develop materials and sponsor workshops so that strengths and weaknesses in student learning can be focused upon.

Dissemination activities are intended to focus attention on the knowledge, skills, and related conceptual areas where needs have been identified. Statewide workshops, the convening of task forces, and materials development are examples of follow-up activities that have resulted directly from assessments.

It should be clear that a testing and measurement unit cannot, in isolation, directly translate results into practical instructional changes. Rather, the key to effective utilization of processes and data is in the establishment of close working relationships with various agencies and educational decision makers within the state. The role of the assessment program in this regard is facilitative.

PIGGYBACK PROGRAM

An important and interrelated aspect of assessment activities is the Local Assessment Option, which is commonly referred to as the "Piggyback

Program". Materials developed for the statewide assessment program are made available through the "Piggyback Program" to school districts that wish to conduct a local assessment program. As part of the "Piggyback" process, local districts are encouraged to establish their own performance expectations for the purpose of comparing results against such expectations. Districts choosing the "Piggyback" option are also able to compare the performance of their students with students in districts of a similar size and type, and with all Minnesota students. With these data, local school districts in Minnesota have useful and reliable information for analyzing their curriculum strengths and needs.

CHAPTER 2

AN OVERVIEW OF MATHEMATICS PERFORMANCE GRADES 4, 8, AND 11

CHAPTER 2

AN OVERVIEW OF MATHEMATICS PERFORMANCE GRADES 4, 8, AND 11

INTRODUCTION

In the 1978-79 Minnesota Educational Assessment in Mathematics, a total of 27,058 students were tested at grades 4, 8, and 11. This chapter provides an overview of results across the grade levels tested and includes recommendations in light of results. Since a similar assessment was conducted in 1974, trend data are also included.

PERFORMANCE ON MATHEMATICS CLUSTERS

Initially, data from the assessment were analyzed according to two logical groupings of exercises: objectives and clusters. In developing the tests, exercises were developed to measure performance on objectives, but for purposes of additional analysis, mathematics educators grouped items into clusters which reflect more common mathematics curricular designations.

Because of the large number of mathematics objectives and related subobjectives, relevant discussion is contained in Chapters 3, 4, and 5. A more complete description of objective and subobjective content is contained in the appropriate grade level appendix section. Table 1 displays student performance across grade levels on mathematics clusters.

Criteria for judging student performance on clusters were established in the same manner as criteria for judging student performance on objectives and subobjectives, described in Chapter 1. For the mathematics assessment, separate groups of educators were used to establish criteria for student performance on subobjectives and clusters, with the same five point scale utilized (strength, potential strength, acceptable, potential need, need) in each instance.

TABLE 1
ACROSS GRADE PERFORMANCE ON MATHEMATICS CLUSTERS

Cluster/Description		Grade 4 Evaluation	Grade 8 Evaluation	Grade 11 Evaluation
B1	Basic Facts of Arithmetic	Strength	Strength	Strength
C1	Computation: Whole Numbers	Potential Strength	Potential Strength	Acceptable
C2	Computation: Fractions	----	Acceptable	Potential Strength
C3	Computation: Decimals	----	Acceptable	Acceptable
C4	Computation: Integers	----	Need	Strength
D1	Terms & Symbols	Potential Strength	Acceptable	----
E1	Estimation	----	Need	----
F1	Introduction to Fractions	Potential Strength	----	----
A1	Algebraic Expressions	----	Acceptable	Potential Strength
A2	Algebraic Applications	----	Acceptable	Acceptable
G1	Recognition/Geometry	Potential Strength	Need	Potential Strength
G2	Geometric Applications	Strength	Need	Acceptable
I1	Graphs, Maps, Tables	----	Strength	Acceptable
M1	Measurement Systems	Potential Strength	Acceptable	Strength
MM1	Metric Measurement	Strength	Need	Acceptable
P1	Properties of Numbers	Potential Strength	Potential Need	Potential Need
PS1	Basic Problem Solving	Strength	Acceptable	Potential Strength
PS2	Advanced Problem Solving	Strength	Strength	Potential Strength
PS3	Consumer Problem Solving	----	----	Need
SP1	Probability and Statistics	----	----	Need
T1	Trigonometry	----	----	Potential Need

Summary

In Table 1, which displays student performance against established criteria across grade levels, the evaluations by subject specialists show considerable strength across grade levels, especially in areas which can be considered "basic", or computational.

The overall meaning and implications of these results can best be determined by interpretations included in individual grade level chapters contained in this report.

MINNESOTA AND NATIONAL PERFORMANCE COMPARISONS

In the development of assessment instruments, items which meet Minnesota objectives are selected from the National Assessment of Educational Progress (NAEP). Thus, it is possible to compare Minnesota student performance with students from across the nation and in the Central United States.[1] Such comparisons provide insights into how well Minnesota students are doing compared to others. Table 2A displays the overall percent correct for Minnesota and national students on identical items in 1978, while Table 2B displays 1974 and 1978 Minnesota and national performance comparisons.

---

[1] The Central United States as defined by NAEP, consists of Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, and Wisconsin.

TABLE 2A
1978 MINNESOTA AND NATIONAL COMPARISONS
GRADES 4, 8, 11

	Overall Percent Correct		
Student Groups	Grade 4	Grade 8	Grade 11
Minnesota	75.6%	60.4%	63.6%
National	70.0%	56.1%	59.8%
Central U.S.	73.0%	59.6%	62.1%

TABLE 2B
1974 AND 1978 NATIONAL PERFORMANCE COMPARED TO MINNESOTA PERFORMANCE

Student Group	Grade 4		Grade 8		Grade 11	
	1974	1978	1974	1978	1974	1978
National	-5.4%	-5.6%	-5.5%	-4.3%	-3.3%	-3.8%
Central U.S.	-3.0%	-2.6%	-2.6%	-0.8%	-1.6%	-1.5%

Summary

As can be observed in the data displayed in Tables 2A and 2B, Minnesota performance compares favorably with both the Central U.S. and the nation and this advantage has generally been maintained across the grade levels from 1974 to 1978. There has been a slight relative decline compared with the nation (1.2%) and with the Central U.S. (1.8%) at the eighth grade level, which is a concern in that a trend may be emerging, but overall, Minnesota students continue to maintain a high level of performance relative to their counterparts in the nation and the Central U.S., a very positive finding.

PERFORMANCE BY SCHOOL DISTRICT CATEGORY

In developing the sample for the mathematics assessment, each public school district in the state was assigned to one of five categories, based on district size and type. From these assignments, students were randomly selected for testing. The following district categories were identified for the sample:

Category 1: Cities of the First Class - Duluth, Minneapolis, St. Paul.

Category 2: Seven-county Metropolitan Area (Developmental Region 11), excluding districts in Category 1.

Districts not included in Categories 1 and 2 with the following K-12 enrollments:

Category 3: 2,000 and Over Students.

Category 4: 1,000-1,999 Students.

Category 5: 1-999 Students.

An analysis of student performance by school district category, as displayed in Table 3, indicates that there is relatively little difference in performance at grades four and eight, except in District Category 1 (Cities of the First Class), where there is a relative major disadvantage.

However, at grade eleven this difference virtually disappears, and student performance is very close to the state average.

TABLE 3
ACROSS GRADE STUDENT PERFORMANCE
BY SCHOOL DISTRICT CATEGORY

Grade	All Minnesota Students	School District Category 1 Cities of 1st Class	2 7-county Metro Area	3 2,000+ Students	4 1,000-1,999	5 1-999 Students
4	77.1%	70.4%	78.8%	78.5%	77.0%	76.3%
8	65.4%	55.5%	68.[illegible]	67.1%	63.2%	65.3%
11	64.1%	62.7%	64.6%	65.9%	64.0%	62.5%

This dramatic relative improvement in student performance by students in District Category 1 has occurred only once before, in the eleventh grade reading assessment. Among other factors, this could, in part, be due to an increased emphasis on both reading and mathematics at the secondary level in those districts. The change in student performance should however, be the subject of further analysis.

STUDENT PERFORMANCE BY SEX

Table 4 displays the performance of girls relative to boys. On overall performance, girls display a very slight advantage at grade four, which declines to a slight disadvantage at grade eleven. A possible explanation for the deficit is that fewer girls than boys take mathematics courses in high school. However, the actual reasons and implications of such a deficit cannot be determined from the data and further study is suggested.

TABLE 4
GIRLS PERFORMANCE RELATIVE TO BOYS

Grade 4	Grade 8	Grade 11
+0.8	Identical	-3.3%

1974-1978 COMPARISON ON IDENTICAL ITEMS

Table 5 displays a comparison of student performance on identical items on the 1974 and 1978 assessments. An analysis of Table 5 indicates that there was a small overall increase in performance at grade four and a small decrease at grades eight and eleven. However, the interesting data emerge when results are analyzed by category content. Generally, there has been a relative increase in the area of arithmetic skills, while there has been a decline in the area of problem solving, or application. The increase in arithmetic skills is most pronounced at grade four, and to a lesser extent at grade eight. The decrease in problem solving, or application skills is most pronounced at grade eleven, and to a lesser extent at grade eight.

TABLE 5
1974 AND 1978 COMPARISONS ON IDENTICAL ITEMS

Grade	Overall Performance	
	1974	1978
4	74.4%	75.6%
8	72.2%	70.9%
11	52.9%	51.6%

This same trend was evident in the National Assessment of Educational Progress (NAEP) data and, as a result, it is reasonable to point out that an important trend exists; that of students becoming more proficient in the arithmetic skills areas and less proficient in the higher level application skills. It is suggested that this trend, along with resultant implications receive further study and analysis.

PERFORMANCE RELATIVE TO SELECTED VARIABLES

Table 6 displays a comparison of overall performance on the assessment related to selected variables. Relative to the use of calculators in the classroom, greater use at the fourth grade level is associated with lower overall performance on the assessment. A trend is not so clear at the eighth grade level, but at the eleventh grade level greater calculator use is associated with higher overall performance. It would seem that this issue needs more study as the role of calculators in education is more clearly defined.

Students at the fourth and eighth grade levels who reported having studied the metric system had higher overall performance than students who reported they had not studied the metric system. The question was not asked at the eleventh grade level.

There was a direct correlation between students' feelings about math and their overall performance on the assessment. The more students reported liking math, the higher their overall performance. Likewise, there was also a direct correlation between students' self-appraisal and their performance on the assessment.

These data suggest that there are many factors that influence students' performance relative to mathematics. Some factors are more easily influenced by the education system than others, but these, and others as well need to be taken into account whenever performance factors are considered.

TABLE 6
COMPARISONS OF STUDENT PERFORMANCE ACROSS GRADE LEVELS ON SELECTED VARIABLES

	OVERALL PERFORMANCE ON ASSESSMENT								
Reporting Variable	No. of Students	% of Students	Grade 4	No. of Students	% of Students	Grade 8	No. of Students	% of Students	Grade 11
Use of Calculators:									
Not at all	5542[1]	74%[2]	78.7%	4902[1]	60%[2]	64.7%	789[1]	17%[2]	58.8%
Once in awhile	1741	23%	76.8%	2583	32%	66.2%	2483	54%	65.3%
For everything except tests	133	2%	69.5%	525	6%	65.6%	873	19%	66.9%
For everything	69	1%	59.5%	116	1%	61.1%	417	9%	69.3%
Study Metric System:									
No	1815	23%	74.6%	1527	19%	59.3%	----	---	-----
Yes	4938	62%	80.3%	6540	81%	67.0%	----	---	-----
I don't know	1272	16%	72.9%	----[3]	---	-----	----	---	-----
Feelings About Math:									
Don't like math at all	----	---	-----	697	8%	54.5%	----	---	-----
Least favorite subject	----	---	-----	713	9%	57.3%	938	20%	54.6%
Not one of favorites	1266	16%	73.9%	2839	35%	65.4%	1901	41%	61.8%
Math is o.k.	3180	39%	77.7%	----	---	-----	----	---	-----
One of favorites	----	---	-----	3295	40%	68.8%	1524	33%	72.8%
Favorite subject	3649	45%	79.1%	666	8%	68.5%	231	5%	77.9%
Self Appraisal: (How smart are you in relation to others)									
At the bottom	----	---	-----	144	2%	46.4%	----	---	-----
Well below average	----	---	-----	212	3%	52.1%	----	---	-----
Slightly below average	----	---	-----	783	9%	53.0%	----	---	-----
Not as smart	1022	13%	67.0%	----	---	-----	546	12%	49.9%
Average	----	---	-----	3332	40%	61.2%	----	---	-----
About as smart	6067	75%	71.9%	----	---	-----	2951	65%	63.2%
Slightly above average	----	---	-----	2954	35%	71.8%	----	---	-----
Smarter than others	969	12%	83.8%	----	---	-----	1031	23%	76.8%
Well above average	----	---	-----	924	11%	77.2%	----	---	-----

[1] Indicates number of students responding to question

[2] Indicates percent of total responses within category

[3] A blank indicates phrase or question not used at that grade level

FUTURE DIRECTIONS

In analyzing student performance on the assessment, and in correlating performance with selected variables, some directions seem evident. Foremost of these is the need to place greater emphasis on teaching problem solving and the application of basic mathematics skills to day-to-day problems. This emphasis is also consistent with recommendations of the National Council of Teachers of Mathematics in their publication, "An Agenda for Action," (1980)[1] which outlines suggested curriculum priorities for the coming decade.

Of the five categories of school districts in the eleventh grade sample, students from small high schools (1-999) had a somewhat lower overall performance than students in other district categories. Coupled with information that many of the small schools are not offering complete academic math programs each year, it is suggested that attention is needed to determine if a trend is developing.

In looking toward the next mathematics assessment, there is a need to develop ways to test problem solving more effectively. The use of multiple choice items does not seem to readily lend itself to accurate testing in this area. Another area that will be explored for possible incorporation in the next assessment is a process for testing students on the use of the calculator, both for computation and for problem solving. If persons reading this report have ideas on how these two efforts can be expedited, please communicate them to the mathematics consultant, Minnesota Department of Education. Your assistance will be greatly appreciated.

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[1] The National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia 22091, 1980.

Assessment results indicate that there are other areas of the mathematics curriculum where additional attention could be focused, including percent, geometry, metrics, consumer problems, and probability and statistics. Activities related to program development in these areas will be brought to the attention of teachers in the immediate future.

CHAPTER 3

FOURTH GRADE ASSESSMENT

CHAPTER 3
FOURTH GRADE ASSESSMENT

INTRODUCTION

The fourth grade mathematics assessment was conducted in February, 1979, and included 8,337 students, or about 15% of the Minnesota fourth grade public school population. Students were selected from five categories of school districts by means of a random sampling plan. The size of the sample and the random nature of the selection means that generalizations from the data can be made to the total population of fourth graders with a high degree of confidence.

PROCEDURES

Instrument Development

The instruments in the Minnesota Educational Assessment Program are criterion-referenced. That is, questions are developed to measure student performance relative to specified goals, objectives, and subobjectives. The primary source for item development is the publication, Mathematics Objectives, Fourth Grade (1978) which was developed in the Division of Instruction (see Appendix, Section A4, page 101). Results are essentially reported against these goals, objectives, and subobjectives.

For the mathematics assessment, the following five major areas were addressed across grade levels:

1. RECALLING AND/OR RECOGNIZING DEFINITIONS, FACTS, AND SYMBOLS.

   A task assessing this ability requires that the student be able to recognize typical mathematical symbolism, or to recall specific facts.

2. PERFORMING MATHEMATICAL MANIPULATIONS.

   This includes tasks which require the student to carry out single operations and procedures (or sequences of these) that have been previously learned and are specifically requested.

3. UNDERSTANDING MATHEMATICAL CONCEPTS AND PROCESSES.

   This level includes tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another.

4. SOLVING MATHEMATICAL PROBLEMS--SOCIAL, TECHNICAL, AND ACADEMIC.

   Assessment of the ability to solve problems requires the student to demonstrate the ability to select knowledge, skills, information, and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

5. USING MATHEMATICS AND MATHEMATICAL REASONING TO ANALYZE PROBLEM SITUATIONS, DEFINE PROBLEMS, FORMULATE HYPOTHESES, MAKE DECISIONS, AND VERIFY RESULTS.

   This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Because several subobjectives fall into categories considered consistent with common curricular designations, an additional clustering of exercises was completed by a committee of mathematics educators. Each of these clusters incorporates several subobjectives and provides a sufficient number of exercises to allow maximum generalizability of results. Eleven clusters were identified for fourth grade students.

In addition, of the 171 items in the assessment, 50 (29%) were selected from the National Assessment of Educational Progress (NAEP) in order to provide comparisons with national groups of students. The remaining items were developed by subject specialists working with Department of Education personnel.

REPORTING OF DATA

In the following sections, data from the assessment are reported in a variety of formats in order to provide a comprehensive view of Minnesota student performance in Mathematics.

PERFORMANCE ON SUBOBJECTIVES AND CLUSTERS

Subobjectives

In this section, student performance is analyzed according to two logical groupings of exercises: subobjectives and clusters. While grouping exercises by subobjectives provides one level of analysis, an additional grouping of exercises was deemed essential since clusters reflect more common mathematics curricular designations. The Roman numeral preceeding each subobjective (Table 7) identifies the goal to which it is assigned. The five goals reflect an ascending order of cognitive difficulty, with Roman I representing the lowest level (recall, recognition) of cognitive classification and Roman V (reasoning, analysis) representing the highest.

A summary of student performance on subobjectives with evaluations by subject specialists is displayed in Table 7. A more complete table of student performance on subobjectives is contained in the Appendix (Section A1, page 95.

TABLE 7
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 4

EVALUATION	SUBOBJECTIVES
Strength	IA1, IA2, IA3, IA4, IB1, IB2, ID1, IIA1b, IIC1, IIC2, IIC3, IIC8, IID1, IID5, IIIA6, IIIA9, IIIC4, IIC6, IIIC13, IIID1, IVA1, IVC1, VA1, VD2
Potential Strength	IC1, ID2, ID3, IE1, IIA2, IIA3, IIA6, IIA8, IIA9, IIC5, IIC6, IIIA3, IIIA5, IIIA8, IIIA10, IIIC1, IIIC2, IIIC3, IIIC5, IIIC12, IIIC17, IV 3, VA2, VA4, VB2
Acceptable	IB5, IIA5, IIC4
Potential Need	IIA1a, IIIC16
Need	IIA4

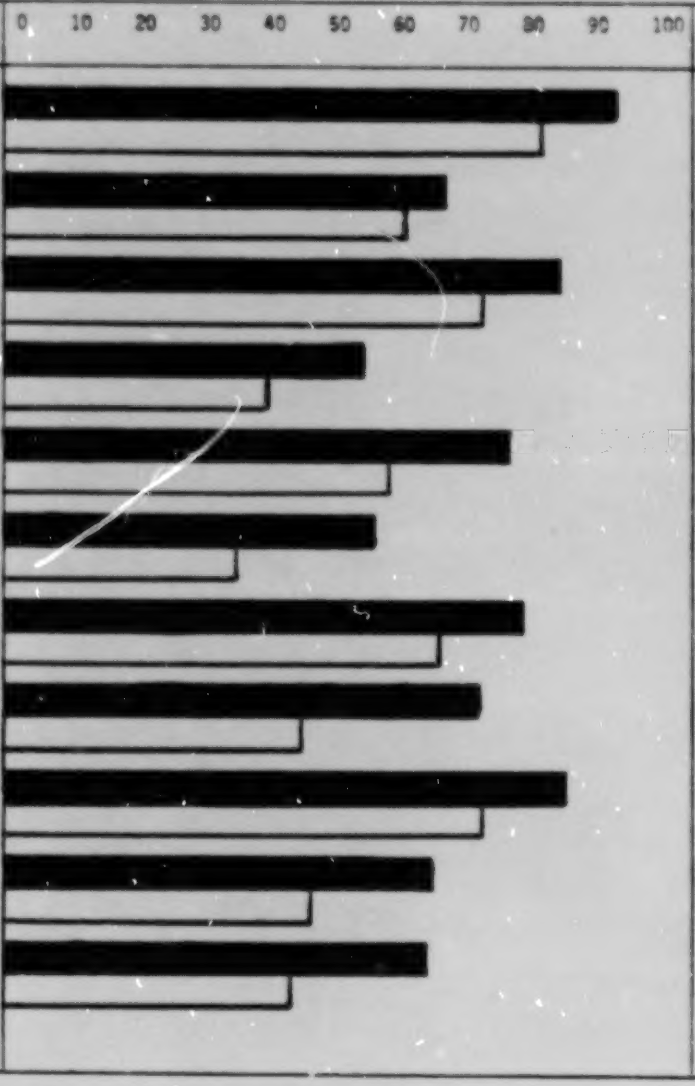
Student performance on clusters, with evaluations by subject specialists is displayed in Table 8. The definition and general content of clusters is contained in the Appendix (Section A2, page 97).

Criteria for student performance on clusters were established by mathematics educators in the same manner as criteria for student performance on subobjectives described earlier. Different committees of educators were used for establishing criteria for subobjectives and clusters, thereby allowing different groups to judge assessment results. The same five point scale (strength, potential strength, acceptable, potential need, need) for evaluating performance was used by each group.

The darker bars on the graph in Table 8 represent actual student performance on clusters, while the lighter bars represent the minimum level of performance considered acceptable by subject specialists.

TABLE 8
STUDENT PERFORMANCE ON MATHEMATICS CLUSTERS
GRADE 4

Cluster/Descriptor	Actual Perf. Criteria*	Evaluation*
B1 - Basic Facts of Arithmetic	92% 81%*	Strength
C1 - Computation With Whole Numbers	66% 60%*	Potential Strength
D1 - Terms and Symbols	84% 72%*	Potential Strength
F1 - Introduction to Fractions	53% 38%*	Potential Strength
G1 - Geometric Properties	76% 59%*	Potential Strength
G2 - Applications of Geometry	56% 33%*	Strength
M1 - Measurement Systems	79% 65%*	Potential Strength
MM1 - Metric Measurement	71% 45%*	Strength
P1 - Properties of Numbers	86% 71%*	Potential Strength
PS1 - Basic Problem Solving	65% 45%*	Strength
PS2 - Patterns, Logic, Advanced Problem Solving	64% 42%*	Strength



*Criteria and the evaluation of student performance were determined by subject specialists.

Dark bars represent actual performance. Light bars represent criterion levels established for acceptable performance.

Summary of Performance - Subobjectives and Clusters

As can be seen by the data displayed in Table 7, relative to criteria established by mathematics educators, student performance on subobjectives was very positive. On 55 subobjectives, student performance was considered to be a strength or a potential strength on 49 (89%). On three subobjectives (5%) dealing with skills in division, adding whole numbers, and the checking process in subtraction, student performance was below a level considered acceptable by mathematics educators. On three subobjectives (5%), student performance was considered to be acceptable, which means that performance was within three percent of the single criterion point established by subject specialists.

An analysis of student performance on clusters (Table 8) indicates a consistency with the high level of performance observed on subobjectives (Table 7). On every cluster, performance was above the minimum level established by mathematics educators. However, in relation to criteria established by mathematics educators, there was considerable variation in student performance on exercises dealing with the four basic mathematical operations in Cluster C-1, *Computation With Whole Numbers*. Student performance was considered a strength in addition and multiplication, a potential strength in subtraction, and a need was identified in division. Overall average performance for the cluster was judged a potential strength.

Given the performance of students on subobjectives and clusters against criteria established by mathematics educators, there appears to be a definite need to focus subsequent developmental efforts at this grade level in the area of division. Student performance on subobjectives indicated a potential need in the areas of adding whole numbers and in the checking, or addition process in subtraction. A potential need indicates that no clear need is present, but it does suggest that those areas receive additional attention from mathematics educators as corrective and developmental plans and activities are initiated.

COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE ON SELECTED ITEMS

In identifying and developing items for each assessment, an effort is made to include quality items from the National Assessment of Educational Progress (NAEP) that also reflect the goals and objectives of Minnesota educators. The inclusion of NAEP items allows for comparisons with national and Central United States[1] groups of students.

Of the 171 items in the assessment, 50 (29%) were from the NAEP program. Minnesota and national performance comparisons are displayed in Table 9.

Summary

In comparison to national students on the 50 items, Minnesota students scored higher on 31 items (62%), lower on 9 items (18%), and there was no significant difference on 10 items (20%).

In comparison to Central United States students, Minnesota students scored higher on 16 items (32%), lower on 11 items (22%), and there was no significant difference on 23 items (46%).

Minnesota students had an overall average score of 75.6% on the 50 NAEP items, compared to 70.0% and 73.0% for national and Central United States students respectively. Thus, in relation to both national and Central United States students, Minnesota students displayed a performance advantage.

In comparison to national students, Minnesota students showed relative strength on items related to subobjectives IA1, *Number Facts - Addition*, IA2, *Number Facts - Subtraction*, IA3, *Number Facts - Multiplication*, and IIIA1, *Solving Word Problems*. In the criteria setting process where student

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[1] The Central United States, as defined by NAEP, consists of Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, and Wisconsin.

TABLE 9
COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE
GRADE 4

SUBOBJECTIVE/DESCRIPTOR		ITEM NUMBER	MN AV. PERCENT CORRECT	NAT'L AV. PERCENT CORRECT	MN/NAT'L PERFORMANCE DIFFERENCE*	CENTRAL U.S. AV. PERCENT CORRECT	MN/CENTRAL U.S. PERFORMANCE DIFFERENCE*
IA1	Number Facts-Addition	1-1	96.7%	95.0%	H	95.8%	NSD
IA1	Number Facts-Addition	1-2	97.1%	95.6%	H	97.3%	NSD
IA1	Number Facts-Addition	1-3	98.3%	94.9%	H	97.6%	NSD
IA1	Number Facts-Addition	1-4	96.5%	91.9%	H	94.3%	H
IA1	Number Facts-Addition	1-5	97.3%	92.6%	H	96.1%	NSD
IA1	Number Facts-Addition	1-6	97.5%	92.6%	H	95.5%	H
IA2	Number Facts-Subtraction	2-1	96.6%	92.9%	H	94.6%	NSD
IA2	Number Facts-Subtraction	2-2	96.3%	88.6%	H	90.6%	H
IA2	Number Facts-Subtraction	2-3	97.5%	92.2%	H	94.0%	H
IA2	Number Facts-Subtraction	2-4	91.8%	79.2%	H	83.8%	H
IA2	Number Facts-Subtraction	2-5	97.0%	85.0%	H	88.8%	H
IA2	Number Facts-Subtraction	2-6	95.6%	81.3%	H	87.1%	H
IA3	Number Facts-Multiply.	3-1	94.7%	88.6%	H	90.4%	H
IA3	Number Facts-Multiply.	3-2	98.0%	94.5%	H	96.2%	NSD
IA3	Number Facts-Multiply.	3-4	93.1%	82.6%	H	87.5%	H
IA3	Number Facts-Multiply.	3-5	88.0%	75.4%	H	78.9%	H
IB2	Numeration-Symbols	2-16	94.9%	92.1%	NSD	91.5%	NSD
ID3	Identify Coins	4-23	85.7%	88.6%	L	89.1%	NSD
ID3	Identify Coins	4-24	85.5%	88.8%	L	89.3%	L
IE1	Recognize Symbols	2-27	51.9%	45.1%	H	49.1%	NSD
IE1	Recognize Symbols	2-28	98.3%	87.8%	H	90.2%	H
IE1	Recognize Symbols	4-20	79.6%	75.7%	H	81.8%	NSD
IIA1a	Add Whole Numbers	1-13	70.9%	79.9%	L	85.1%	L
IIA1a	Add Whole Numbers	2-10	90.3%	90.6%	NSD	90.4%	NSD
IIA1a	Add Whole Numbers	2-11	87.0%	81.9%	H	85.3%	NSD
IIA2	Subtract Whole Numbers	2-12	47.0%	33.9%	H	44.1%	NSD
IIA3	Multiply Whole Numbers	1-14	68.2%	48.3%	H	45.9%	H
IIA9	Count By 2, 3, 5, 10	2-33	89.9%	85.4%	H	85.7%	H
IIA9	Count By 2, 3, 5, 10	3-27	86.0%	86.9%	NSD	90.3%	L
IIC4	Read Temperature	4-21	81.0%	80.8%	NSD	84.4%	NSD
IID5	Choose Graph	2-35	44.3%	32.2%	H	36.1%	NSD
IIIA10	Select No Sentences	2-34	72.7%	55.8%	H	61.1%	H
IVA1	Solve Word Problem	1-19	56.1%	33.2%	H	33.0%	H
IVA1	Solve Word Problem	1-28	32.9%	15.3%	H	19.1%	H
IVA1	Solve Word Problem	1-31	65.1%	59.1%	H	64.8%	NSD
IVA1	Solve Word Problem	1-35	40.6%	9.1%	H	11.4%	H
IVA1	Solve Word Problem	2-24	75.0%	77.7%	NSD	86.3%	L
IVA1	Solve Word Problem	2-25	70.7%	86.1%	L	88.1%	L
IVA1	Solve Word Problem	2-26	36.3%	40.2%	L	42.8%	NSD
IVA1	Solve Word Problem	2-29	89.6%	83.6%	H	90.0%	NSD
IVA1	Solve Word Problem	3-17	5.2%	3.1%	H	3.3%	NSD
IVA1	Solve Word Problem	3-23	79.6%	74.3%	H	77.2%	NSD
IVA1	Solve Word Problem	3-25	37.5%	35.8%	NSD	38.5%	NSD
IVA1	Solve Word Problem	4-13	53.1%	55.6%	NSD	57.2%	NSD
IVA1	Solve Word Problem	4-15	87.0%	85.4%	NSD	89.8%	NSD
IVA1	Solve Word Problem	4-16	14.2%	8.0%	H	5.8%	H
IVB3	Find Volume	2-19	58.0%	78.7%	L	81.3%	L
IVB3	Find Volume	2-20	44.1%	38.5%	H	42.4%	NSD
VA1	Estimate Answer	3-20	74.7%	77.2%	L	81.0%	L
VB2	Solve Problem	4-26	86.2%	62.8%	H	68.3%	H
OVERALL AVERAGE			75.6%	70.0%		73.0%	

Key: H - Minnesota significantly higher than comparison group performance.
L Minnesota significantly lower than comparison group performance.
NSD - No statistically significant difference between scores.

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*Computed at .05 level of significance.

performance is judged against standards established by mathematics educators, student performance on those subobjectives was considered a strength.

In comparison to Central United States students, Minnesota students showed relative strength on items related to subobjectives IA2, *Number Facts - Subtraction*, and IA3, *Number Facts - Multiplication*.

Items where Minnesota students scored relatively poorly compared to either national or Central United States students were scattered and no patterns of need emerged. Minnesota students scored below both national and Central United States students on two items from subobjective ID3, *Identifying Coins*, but as displayed in Table 7, overall performance on the subobjective was 91.8%, and performance was considered a potential strength by mathematics educators.

Overall, Minnesota fourth grade student performance is quite positive when compared to national and Central United States students.

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STUDENT PERFORMANCE IN CATEGORIES OF DISTRICTS

In the assessment, the total sample of fourth grade students was selected proportionally from the following categories of school districts:

Category 1: Cities of the First Class - Duluth, Minneapolis, St. Paul.

Category 2: Seven-county Metropolitan Area (Developmental Region 11), excluding districts in Category 1.

Districts not included in Categories 1 and 2 with the following K-12 enrollments:

Category 3: 2,000 and Over Students.

Category 4: 1,000-1,999 Students.

Category 5: 1-999 Students.

Table 10 displays the average performance of all Minnesota students as well as the average performance of students by school district category.

TABLE 10
STUDENT PERFORMANCE BY SCHOOL DISTRICT CATEGORY
GRADE 4

All Minnesota Students	1 Cities of 1st Class	2 7-county Metro Area	3 2,000+ Students	4 1,000-1,999	5 1-999 Students
77.1%	70.4%	78.8%	78.5%	77.0%	76.3%

Summary

When comparing student performance by school district category, the data indicate that the only major deviation from the Minnesota average was evidenced in District Category 1 (Cities of the First Class) with students here scoring 6.7% below the state average. Students in District Category 4 (1,000-1,999 Students) scored almost at the state average. Students in District Categories 2 (Seven-county Metro Area) and 3 (2,000 and Over Students) scored slightly above the state average, while students in District Category 5 (1-999 Students) scored slightly below the state average.

The relatively low performance by students in one school district category should be a cause for concern, and this phenomenon should be the focus of additional study and related activities.

THE RELATIONSHIP BETWEEN REPORTING VARIABLES AND MATHEMATICS PERFORMANCE

A student information section of the assessment requested that participants respond to questions relating to their perceptions, attitudes, and background. Student responses were then compared to overall performance on the assessment. For example, students who reported they had not studied the metric system had an overall average score of 74.6%, compared to an overall average score of 80.3% for students who reported they had studied the metric system.

Table 11 displays a summary of the student information section along with overall performance scores. The complete student information section, with overall performance scores is contained in the Appendix (Section A3, page 99).

Summary

When the data from the student information section were compared with overall performance on the assessment, the following conclusions were evident:

1. Girls displayed a slight performance advantage over boys on the assessment.
2. Increased student usage of calculators was directly related to lower performance on the assessment.
3. Students who reported studying the metric system had considerably higher performance on the assessment than students who reported not studying metrics.
4. Increased use of manipulatives was related to higher performance to a point. Performance dropped sharply for students reporting using many (10-15) manipulatives.
5. Attitudes toward math and attitudes toward self were directly related to performance; the more students reported liking mathematics, and the higher their perception of their ability, the higher their overall performance on the assessment.

Considering the data from the fourth grade assessment in mathematics, it is possible to generate a composite picture of a participant with the potential of high performance on a majority of mathematics objectives. It

TABLE 11
COMPARISON OF STUDENT RESPONSES ON QUESTIONS TO THEIR OVERALL TEST SCORES
GRADE 4

REPORTING VARIABLE	OVERALL PERFORMANCE
Sex:	
Boys	77.0%
Girls	77.8%
Use of Calculators:	
Not at all	78.7%
Only once in awhile	76.8%
Everything except tests	69.5%
Everything	59.5%
Study Metric System in School:	
No	74.6%
Yes	80.3%
Don't know	72.9%
You Compared to Others:	
Not as smart	67.0%
About as smart	71.9%
Smarter	83.8%
Math Ability:	
Poor	63.2%
Fair	72.0%
Good	78.6%
Excellent	81.9%
Feelings About Math:	
Not among favorites	73.9%
Math is O.K.	77.7%
Favorite subject	79.1%
Use of Manipulatives:	
None	72.1%
1 - 6	78.1%
7 - 9	79.2%
10 - 15	75.2%

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is also possible to generate a composite picture of a participant with the potential of low performance on a majority of the objectives.

A participant with the potential for high performance on a majority of mathematics objectives:

- Lives in a suburban or large (2,000 and Over) school district;
- Is nine or ten years old;
- Has an above average self-appraisal;
- Has an above average self-appraisal in mathematics ability;
- Includes mathematics among favorite subjects;
- Has studied the metric system;
- Has used some manipulative aids;
- Does not use a calculator in the classroom.

In contrast, a participant with the potential for low performance on a majority of mathematics objectives:

- Lives in one of the first class cities;
- Is one of the younger or older students in class;
- Appraises self as not as smart as others;
- Does not like mathematics;
- Has not studied the metric system;
- Does not use manipulative aids;
- Uses a calculator for everything;
- Has a low self rating in mathematics ability.

As can be seen from the composite pictures, many factors can influence student achievement in mathematics. Some of the factors can be addressed directly by educators, while others are more complex and have their roots either partly or wholly in the greater society in which we live. But the implications of these data deserve attention and should provide a focus for planning and relevant action.

MINNESOTA 1974 AND 1978 PERFORMANCE COMPARISONS

Discussed in this section are those meaningful comparisons which can be made between mathematics performance in 1974 and 1978. In those instances where changes in program procedures limit direct comparability between 1974 and 1978 performance, the nature of limitations in comparability will be noted. Discussion in this section will focus on the following four comparisons: 1) performance on identical items; 2) performance relative to national groups; 3) performance of boys and girls; and 4) performance on clusters.

Performance on Identical Items

In 1974, the assessment was conducted utilizing nine-year-old students, while in 1978, students who were fourth graders participated. In order to provide comparability, 1974 data were re-analyzed to obtain results from fourth grade students only. All other testing procedures were compatible between 1974 and 1978, thus allowing valid comparisons to be made.

Thirty-four items in the 1978 assessment were identical with those administered in 1974. A summary of performance comparisons is displayed in Table 12. As can be seen in the table, on the thirty-four items, the 1978 group significantly outperformed the 1974 group on 7 items (21%), there was no significant difference between the groups on 22 items (65%), and on 5 items (15%), the 1978 group scored significantly lower than the 1974 group. The average performance for the 1978 group was 75.6%, while the average performance for the 1974 group was 74.4%. Thus, the 1978 group displayed a slight performance advantage of 1.2% over the 1974 group.

An increase in performance can be noted in the area of *Mathematical Terms and Symbols* (Cluster D-1), while a slight decrease is suggested in the area of *Basic Problem Solving* (Cluster PS-1). On other items on which

TABLE 12
1974 AND 1978 PERFORMANCE COMPARISONS
GRADE 4

CLUSTER/DESCRIPTOR	PACKAGE ITEM	1978 PERF.	1974 PERF.	1978 vs. 1974 PERFORMANCE*
B1 Basic Facts of Arithmetic	2-7	90.8%	94.6%	L
B1 Basic Facts of Arithmetic	3-8	93.3%	94.6%	NSD
C1 Computation: Whole Numbers	2-13	87.7%	89.8%	NSD
C1 Computation: Whole Numbers	3-10	83.0%	76.2%	H
C1 Computation: Whole Numbers	3-11	91.1%	91.7%	NSD
C1 Computation: Whole Numbers	3-12	77.1%	74.1%	NSD
C1 Computation: Whole Numbers	4-5	50.0%	44.4%	NSD
C1 Computation: Whole Numbers	2-14	82.3%	80.0%	NSD
C1 Computation: Whole Numbers	3-33	57.6%	38.8%	H
D1 Terms and Symbols	4-38	68.1%	72.5%	NSD
D1 Terms and Symbols	4-39	77.3%	76.7%	NSD
D1 Terms and Symbols	4-40	77.8%	78.8%	NSD
D1 Terms and Symbols	4-41	90.1%	92.5%	NSD
D1 Terms and Symbols	1-46	90.4%	80.0%	H
D1 Terms and Symbols	1-47	91.5%	80.4%	H
D1 Terms and Symbols	1-48	94.5%	85.7%	H
D1 Terms and Symbols	3-22	86.0%	88.3%	NSD
D1 Terms and Symbols	2-36	78.5%	79.0%	NSD
F1 Introduction to Fractions	2-18	44.3%	44.4%	NSD
F1 Introduction to Fractions	3-18	45.6%	42.9%	NSD
F1 Introduction to Fractions	4-12	49.6%	53.5%	NSD
G1 Recognition/Geometry	3-24	69.2%	68.0%	NSD
P1 Properties of Numbers	4-25	90.6%	88.0%	NSD
P1 Properties of Numbers	3-35	83.2%	91.2%	NSD
PS1 Basic Problem Solving	3-32	81.3%	88.7%	L
PS1 Basic Problem Solving	4-22	47.4%	55.6%	L
PS1 Basic Problem Solving	1-18	62.0%	48.8%	H
PS2 Advanced Problem Solving	1-24	95.6%	95.6%	NSD
PS2 Advanced Problem Solving	1-25	76.7%	82.1%	L
PS2 Advanced Problem Solving	1-26	96.9%	95.8%	NSD
PS2 Advanced Problem Solving	3-30	95.9%	95.5%	NSD
PS2 Advanced Problem Solving	1-17	92.9%	84.9%	H
PS2 Advanced Problem Solving	2-17	58.1%	66.0%	L
PS2 Advanced Problem Solving	4-29	13.0%	10.0%	NSD
Overall Performance		75.6%	74.4%	

Key: H - Minnesota 1978 significantly higher than Minnesota 1974 group performance.
L - Minnesota 1978 significantly lower than Minnesota 1974 group performance.
NSD - No statistically significant difference between group scores.

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*Computed at .05 level of significance.

increases or decreases can be noted, no clear patterns emerge for discussion purposes.

In summary, the 1978 group showed a slight performance advantage over the 1974 group. On items related to one cluster (D-1), performance increased. On one cluster (PS-1) there appears to be a slight decline, although this is not clear cut. As a result, it is suggested that student performance on *Basic Problem Solving* (PS-1) be given the necessary attention to determine if a need does exist.

Performance Relative to National Groups

The fact that 1974 national data for fourth graders is not available limits the extent to which direct statistical comparisons can be made. However, it is possible to make reasonably valid observations where the data for each group are presented separately, and where the characteristics of the groups are taken into account. Table 13 displays these comparison data.

In 1974, Minnesota nine-year-olds outperformed national and Central United States students by 5.4% and 3.0% respectively. In 1978, Minnesota fourth graders outperformed their national and Central United States counterparts by 5.6% and 2.6% respectively. Neither the slight relative advantage (0.2%) for the 1978 group on national comparisons nor the slight relative disadvantage (0.4%) for this group on Central U.S. comparisons is taken to represent a meaningful fluctuation in scores. The relative advantages identified in 1974 have essentially been maintained over a four-year period and the data can be appropriately interpretated to suggest that Minnesota fourth graders continue to compare favorably to national and regional groups of students in the area of mathematics.

TABLE 13
1974 AND 1978 NATIONAL COMPARISONS
GRADE 4

	1974 Nine-Year-Olds					1978 Fourth Graders				
	NUMBER OF ITEMS	MN ABOVE	MN NO DIFFERENCE	MN BELOW	AVERAGE PERFORMANCE	NUMBER OF ITEMS	MN ABOVE	MN NO DIFFERENCE	MN BELOW	AVERAGE PERFORMANCE
National	31	21	8	2	49.5%	50	35	8	7	70.0%
Central U.S.	31	10	19	2	51.9%	50	19	24	7	73.0%
Minnesota	31	--	--	--	54.9%	50	--	--	--	75.6%

Performance of Boys and Girls

In 1974, the average performance for nine-year-old girls was 62.9%, while the performance for boys was 61.0%. In the 1978 assessment, the average performance for girls was 77.8%, while the performance for boys was 77.0%. Girls and boys continue to perform at the same relative level, and there appears to be no relative advantage or meaningful trends in these data.

Cluster Analysis

Eight of the eleven item clusters in the 1978 assessment were also contained in the 1974 assessment. It should be noted that items within clusters are not necessarily identical, and direct comparisons cannot be made between nine-year-olds (1974 sample) and fourth graders (1978 sample). However, meaningful comparisons can be made between professional judgments across time due to the fact that appropriate criteria were established for the groups tested. Data for the cluster comparisons are displayed in Table 14.

It is apparent from the data that the average student performance as well as the evaluations of subject specialists were higher in 1978 than in 1974. The generally higher performance by 1978 students might be attributed to a change in item difficulty levels, or to the fact that in 1974, approximately one-third of the students tested were third graders. However, these factors were taken into account when criteria were established for student performance, and scores tend to vary in a relatively systematic fashion.

Clusters in the area of *Properties of Numbers* (P1), *Geometric Properties* (G1), *Measurement Systems* (M1), *Basic Problem Solving* (PS1), and *Patterns, Logic, and Advanced Problem Solving* (PS2) were strengths in 1974 as well as in 1978. Student performance went from a need in 1974 to a strength in 1978 in *Introduction to Fractions* (F1). The increased performance in the area of fractions is supported by additional data in this report (see

TABLE 14
1974 AND 1978 CLUSTER COMPARISONS
GRADE 4

	1974 (THIRTEEN-YEAR-OLDS)			1978 (EIGHTH GRADERS)		
	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	EVALUATION*	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	EVALUATION*
C1 Computations/ Whole Numbers	65.0%	65.4%	Acceptable	60.2%	66.2%	Strength
P1 Properties of Numbers	73.0%	85.3%	Strength	71.5%	85.9%	Strength
G1 Recognition/ Geometry	56.0%	72.0%	Strength	59.2%	76.0%	Strength
G2 Geometric Applications	34.0%	30.1%	Acceptable	32.5%	55.7%	Strength
M1 Measurement Systems	40.0%	46.1%	Strength	64.5%	78.5%	Strength
F1 Introduction to Fractions	41.0%	25.8%	Need	38.0%	53.4%	Strength
PS1 Basic Problem Solving	57.0%	60.1%	Strength	45.4%	64.8%	Strength
PS2 Advanced Problem Solving	40.0%	50.0%	Strength	42.5%	63.8%	Strength

*Evaluations for 1978 were collapsed into three categories (strength, acceptable, need) as was done in 1974.

Table 8), and is, in part, attributed to the increased attention and developmental activities initiated as a result of an analysis of 1974 student performance.

Relative to the judgments of subject specialists, there were no clusters where student performance declined from 1974 to 1978.

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ANALYSIS OF ERRORS ON MATHEMATICS EXERCISES

From an analysis of errors on the fourth grade mathematics assessment, the following conclusions seem justified:

1. Subtraction across zero (borrowing) caused difficulties for about one-third of the students in the assessment.
2. The number of students who could correctly recognize an odd or even number declined from the 1974 assessment. This concept has implications for students working with fractions. Fourth grade students should be able to differentiate between odd and even numbers. With some attention, these skills should be relatively easy to strengthen.
3. When asked to give the perimeter of an object, many students incorrectly gave the area or one-half the perimeter.
4. More than half of the fourth grade student participants could not select the mathematics sentence to fit a simple word problem. They could not determine whether the sentence was related to multiplication or addition.
5. Fourth grade students were not able to analyze a problem and select the appropriate data needed to find the solution.
6. Many fourth grade students were not able to logically arrive at a valid conclusion based on given data.
7. Fewer than one-thirdof the fourth grade students were able to divide a two digit number by a single digit divisor. Error responses indicate that students seem to know the related multiplication facts but do not understand the relationship between multiplication and division. There is a definite need to increase student's proficiency in single digit division. This is the area of greatest concern at the fourth grade level.

RECOMMENDATIONS FOR IMPROVING MATHEMATICS INSTRUCTION

The following recommendations are generally suggested by the data, but to some extent have been supplemented by additional observations by selected members of the Minnesota Council of Teachers of Mathematics.

A. Teacher Education

  1. Inservice

    a. Teachers need to upgrade their own process abilities and then develop methods to facilitate students' learning of problem solving techniques. Strategies for teaching "translation" from English to mathematics and vice-versa need to be reinforced and strengthened.

b. Teachers should be able to help students construct charts, graphs and/or tables, and to interpret data presented in any of these forms.

c. The relationship between multiplication facts and division facts needs to be reinforced. Reasons for typical student errors in division need to be examined and corrective teaching strategies reviewed and extended.

d. The inverse relationship between performance and calculator use at the fourth grade level needs to be carefully examined. The indiscriminant and/or inappropriate use of calculators should be discouraged.

e. The use of manipulatives and other techniques and strategies for teaching subtraction across zero (borrowing) need to be reviewed and reinforced.

f. Teachers must recognize the need to teach the value of coins, how to make change for a dollar and how to select the best buy. Student knowledge in these areas cannot be assumed.

g. Awareness of the positive relationship between student performance and use of manipulatives needs to be raised. Techniques for using manipulatives should be reviewed and strengthened.

h. Awareness of the effect of the student's self concept and performance in mathematics needs to be developed. This is particularly important with respect to girls and how they perceive their own mathematical ability.

i. The positive relationship between student performance and knoweldge of the metric system must be recognized. Skills in teaching the metric system of measurement need to be reinforced and extended.

j. Awareness of fourth grade student confusion about how to determine odd and even numbers needs to be raised. Strategies for teaching odd and even need to be reinforced.

2. Preservice

a. College content courses required for elementary teacher preparation should include at least one course in each of the following areas: 1) theory of arithmetic; 2) informal geometry (including metric measurement); 3) probability and statistics; and 4) consumer buying.

b. College methods courses required for elementary teacher preparation should include the following: 1) the appropriate use of manipulatives in the mathematics classroom; 2) strategies and techniques for teaching basic facts and computation with whole numbers, decimals, and fractions; 3) appropriate use of the calculator in elementary mathematics; 4) methods for providing successful experiences in mathematics for all students, because of the relationship between self concept and performance in mathematics.

B. Curriculum

1. Textbooks

a. Presentation of the following should be considered in the selection process:

1) exercises requiring the use of metric measurements;

2) exercises dealing with consumer buying, making change, and money;

3) clearly presented algorithms for computations;

4) opportunities for constructing and interpreting tables, charts, and graphs;

5) exercises requiring the use of manipulatives;

6) informal geometry and exercises dealing with area perimeter and volume;

7) inclusion of role models for girls and minorities which foster a desire to become proficient in mathematics and to view mathematics as a desirable course of study.

2. Support or supplementary material

a. A variety of manipulative aids should be available for student use.

b. Both enrichment and remedial materials (games, puzzles, etc.) should be available for student use.

3. Technology

a. Computer terminals should be available for student use. Every student should have direct experience with the terminal and should understand what a computer can and cannot do.

b. Calculators should be available for appropriate use by students.

C. School/District-wide Policy

1. A system-wide policy should be formulated regarding the following:

a. Teaching of the metric system of measurement. What is taught at each level needs to be clearly defined.

b. Appropriate use of calculators in the classroom. The appropriate grade level and places in the curriculum where calculator use is especially effective needs to be defined.

c. Non-stereotypic role models for girls and minorities. Fostering an awareness of mathematics related career opportunities available for anyone needs to be done on a system-wide basis.

d. Effective teaching strategies for intelligent consumer buying and the related computations with money amounts need to be developed and implemented.

e. Maintaining and upgrading teachers' mathematical competencies. Opportunities for professional growth and development in mathematics should be provided. Policies should encourage teachers to enroll in university and college courses and to participate in mathematics professional organization meetings, conferences, and workshops.

f. Selection of textbooks. Guidelines for selection need to be developed and implemented. Then the selection should be on a system-wide basis.

D. Minnesota State Education Policy

1. Attention must be focused on the need to strengthen fourth grade students' skill in the division of whole numbers. A plan similar to the one which focused on fractions, which was developed after the 1974-75 assessment, needs to be designed and implemented on a statewide basis. Suggested teaching strategies, manipulative aids, and instructional materials which have been effective in instruction in division needs to be compiled and made available to individual school districts.

2. Guidelines for college mathematics course work required for elementary teacher preparation need to be analyzed for their comprehensiveness and currency in light of present mathematics curricula.

3. Leadership in determining the appropriate role of the calculator in elementary school mathematics classes needs to be exhibited. A position paper containing suggested calculator use and/or non-use at each grade level needs to be developed.

4. Information and data supporting the positive effect the use of manipulatives has on student performance in mathematics needs to be disseminated. Guidelines for selecting manipulatives and suggestions for their use as instructional aids needs to be developed and shared with school systems state-wide.

5. Metric measurement refresher workshops need to be organized throughout the state.

6. Leadership in defining the mathematics component of basic skills education needs to be exhibited. Efforts to improve mathematics teaching/learning need to be coordinated with basic skills education at the state department level.

7. Planning for future state-wide assessments should be an ongoing process. The dramatic improvement in student performance in fraction computation can, in part, be attributed to state department efforts made in response to the results of the 1974 assessment in mathematics. Determining curricular needs, planning corrective procedures, implementing the plan, and evaluating the result is a much more effective and desirable procedure than imposed minimal competency testing. It focuses on student needs and curricular weaknesses rather than on the identification of the minimal level of skill in mathematics.

CHAPTER 4

EIGHTH GRADE ASSESSMENT

CHAPTER 4

EIGHTH GRADE ASSESSMENT

INTRODUCTION

The eighth grade mathematics assessment was conducted in November, 1978. A total of 8,921 students, or about 12% of the eighth grade public school population was included in the assessment. Students were randomly selected from five categories of school districts, based on district size and type. The size of the sample and the random nature of the selection means that generalizations from the data can be made to the total population of eighth graders with a high degree of confidence.

PROCEDURES

Instrument Development

The instruments in the Minnesota Educational Assessment Program are criterion-referenced. That is, questions are developed to measure student performance relative to specified goals, objectives, and subobjectives. The primary source for item development is the publication, Mathematics Objectives, Eighth Grade, 1978 which was developed in the Division of Instruction (see Appendix, Section B6, page 132). Results are essentially reported against these goals, objectives, and subobjectives.

For the mathematics assessment, the following five major areas were addressed across grade levels:

1. RECALLING AND/OR RECOGNIZING DEFINITION, FACTS, AND SYMBOLS.

   A task assessing this ability requires that the student be able to recognize typical mathematical symbolism, or to recall specific facts.

2. PERFORMING MATHEMATICAL MANIPULATIONS.

   This includes tasks which require the student to carry out single operations and procedures (or sequences of these) that have been previously learned and are specifically requested.

3. UNDERSTANDING MATHEMATICAL CONCEPTS AND PROCESSES.

   This level includes tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another.

4. SOLVING MATHEMATICAL PROBLEMS--SOCIAL, TECHNICAL, AND ACADEMIC.

   Assessment of the ability to solve problems requires the student to demonstrate the ability to select knowledge, skills, information, and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

5. USING MATHEMATICS AND MATHEMATICAL REASONING TO ANALYZE PROBLEM SITUATIONS, DEFINE PROBLEMS, FORMULATE HYPOTHESES, MAKE DECISIONS, AND VERIFY RESULTS.

   This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Because several subobjectives fall into categories considered consistent with common curricular designations, an additional clustering of exercises was completed by a committee of mathematics educators for purposes of analysis. Each of these clusters incorporates several subobjectives and provides a sufficient number of exercises to allow maximum generalizability of results. Seventeen clusters were identified for eighth grade students.

In addition, of the 177 items in the assessment, 51 (29%) were selected from the National Assessment of Educational Progress (NAEP) in order to provide comparisons with national groups of students. The remaining items were developed by subject specialists working with Department of Education personnel.

REPORTING OF DATA

In the following sections, data from the assessment are reported in a variety of formats in order to provide as comprehensive an analysis as possible of Minnesota student performance in mathematics.

PERFORMANCE BY SUBOBJECTIVES AND CLUSTERS

Student performance on subobjectives, with evaluations by subject specialists is displayed in Table 15. Because of the large number of mathematics subobjectives, Table 15 displays a summary of student performance. A more complete table is contained in the Appendix (Section B1, page 115). The Roman numeral preceeding each subobjective (Table 15) identifies the goal to which it is assigned. The five goals reflect an ascending order of cognitive difficulty, with Roman I representing the lowest level (recall, recognition) of cognitive classification and Roman V (reasoning, analysis) representing the highest.

TABLE 15
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 8

EVALUATION	SUBOBJECTIVES
Strength	IA, ID3, IVF1
Potential Strength	IB2, IB4, IIA2b, IIA14, IID5, IVA3, VA3, VC2, VG
Acceptable	IC1, IC2, ID1, IE1, IIA1b, IIA1c, IIA2a, IIA2c, IIA3a, IIA3b, IIA4, IIA7, IIB1, IIB3, IIC1, IIIA1, IIIB2, IIID2, IVA1, IVA2, IVA7, IVC5, IVC7, IVD2, IVE1, VA1, VB1, VB2, VD
Potential Need	IB1, ID2, ID4, IIA1a, IIA1d, IIA2e, IIA5, IIA8, IIA9, IIB7, IIC2, IID4, IIIB1, IVA6, IVC4, IVD1, IVE5, VB3, VC3
Need	IB5, IIA6, IIA13, IIA18, IIC4

Student performance on clusters, with evaluations by subject specialists is displayed in Table 16. The definition and general content of the clusters is displayed in the Appendix (Section B2, page 117).

Criteria for acceptable student performance on clusters were established by subject specialists in the same manner as criteria for student performance on subobjectives, described earlier. Different committees of educators were used for the two criteria setting processes, thereby allowing different groups to judge assessment results. The same five point scale utilized in evaluating performance on subobjectives was applied (strength, potential strength, acceptable, potential need, need).

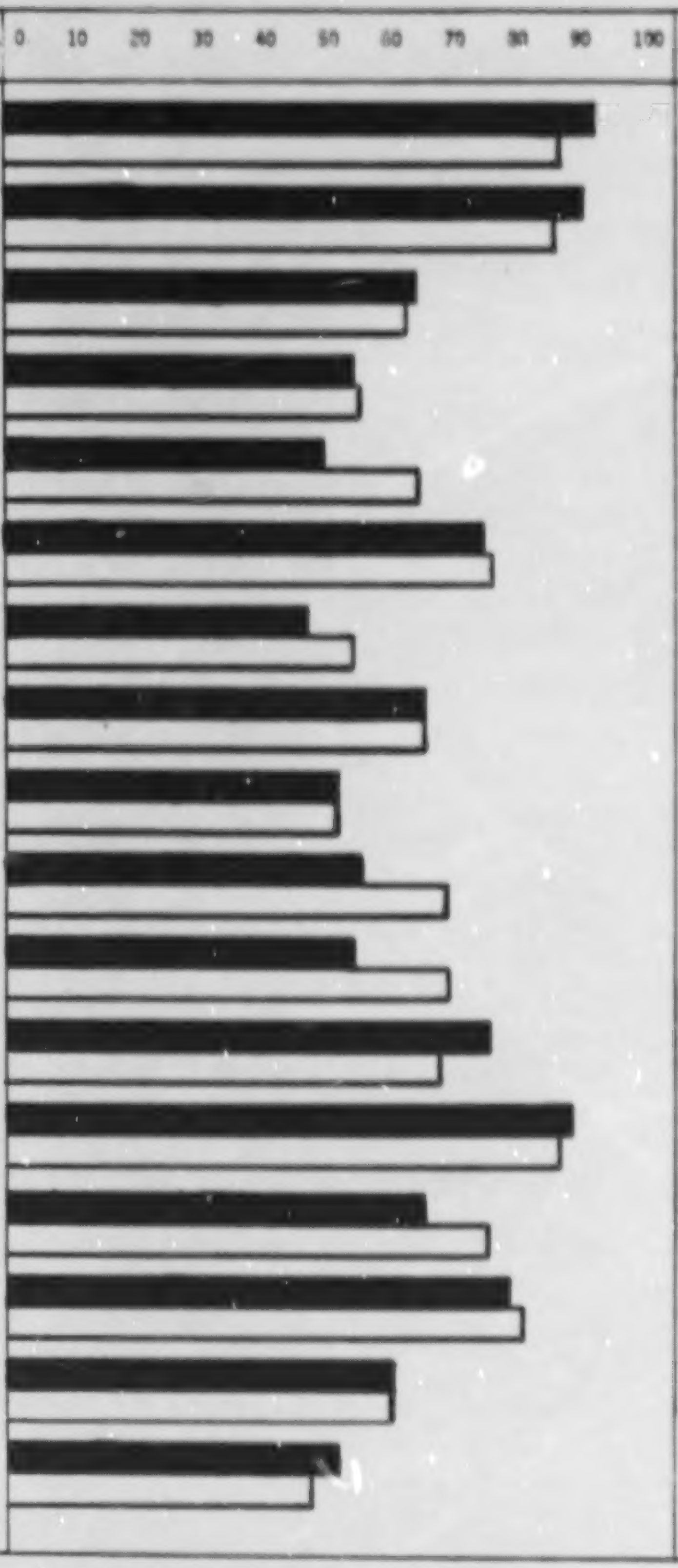
The darker bars on the graph in Table 16 represent the actual student performance for clusters while the lighter bars represent the minimum level for acceptable performance established by subject specialists.

Summary of Performance on Subobjectives and Clusters

An analysis of student performance on subobjectives reveals that on 12 subobjectives (18%) student performance was above an acceptable level while on 24 subobjectives (37%) student performance was classified as a potential need or need. Student performance on subobjectives indicates strength in the areas of number facts, units of money, and map reading. Areas of need were indicated in skills relating to place value, fraction/decimal equivalents, finding percent, ordering fractions, and correcting measurement.

An analysis of student performance on clusters indicates performance above an acceptable level on four clusters (24%) and below an acceptable level on six clusters (35%).

TABLE 16
STUDENT PERFORMANCE ON MATHEMATICS CLUSTERS
GRADE 8



Cluster/Descriptor	Actual Perf. Criteria*	Evaluation
B1 - Basic Facts of Arithmetic	92% 87%*	Strength
C1 - Computation: Whole Numbers	90% 86%*	Potential Strength
C2 - Computation: Fractions	63% 61%*	Acceptable
C3 - Computation: Decimals	53% 54%*	Acceptable
C4 - Computation: Integers	49% 64%*	Need
D1 - Terms and Symbols	74% 75%*	Acceptable
E1 - Estimation	46% 53%*	Need
A1 - Algebraic Expressions	65% 65%*	Acceptable
A2 - Algebraic Applications	51% 51%*	Acceptable
G1 - Recognition Geometry	55% 68%*	Need
G2 - Applications Geometry	54% 69%*	Need
I1 - Graphs, Maps, Tables	74% 67%*	Strength
M1 - Measurement Systems	88% 86%*	Acceptable
MM1 - Metric Measurement	63% 74%*	Need
P1 - Properties of Numbers	76% 79%*	Potential Need
PS1 - Basic Problem Solving	60% 60%*	Acceptable
PS2 - Advanced Problem Solving	51% 47%*	Strength

*Criteria and the evaluation of student performance were determined by subject specialists.

Student performance on clusters indicates strength in the areas of basic facts, computation with whole numbers, graphs, maps, and tables, and advanced problem solving. Clusters where student performance was below an acceptable level included computation with integers, estimation, geometry, metric measurement, and properties of numbers.

Conclusions

As determined by Minnesota subject specialists in mathematics, eighth grade students in Minnesota appear to have acceptable skill levels in computation with whole numbers, computation with fractions, interpretation of graphs, maps, and tables, and problem solving. Of these four areas, computation with fractions and problem solving are central topics in junior high curricula. Therefore, the performance levels dramatize the positive aspects of mathematics achievement in Minnesota schools.

Eighth grade student performance was below an acceptable level in the areas of estimation, geometry, and metric measurement. All of these areas should be an integral part of seventh and eighth grade mathematics programs. Geometrical concepts, as reflected in the assessment, were simple and basic and it would seem that results should have been higher. Estimation skills increase in importance as calculators or mini-computers increase in usage. Students' ability to estimate accurately was not demonstrated by the results. Thus, data on student performance on subobjectives and clusters should help focus developmental activities for increasing students' skills in specific areas in the junior high mathematics curriculum.

COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE ON SELECTED ITEMS

In the development of each assessment instrument, an effort is made to include quality items from the National Assessment of Educational Progress (NAEP) if items also reflect the goals and objectives of Minnesota educators.

The inclusion of NAEP items allows for comparisons with national and Central United States[1] groups of students.

Of the 177 items in the assessment test, 51 (29%) were from the NAEP program. Minnesota and national performance comparisons are displayed in Table 17.

Summary

On the 51 NAEP items on which comparisons were made, Minnesota students scored above their national counterparts on 26 items (51%). There was no significant difference on 16 items (31%), and on 9 items (18%), Minnesota students scored lower than national students. The overall average for Minnesota students was 60.4% compared with 56.1% for the national group.

In analyzing the math subobjectives from which the items were taken, Minnesota student performance showed strength on subobjectives related to computation, reasoning, graphing, and generalization in geometry. Minnesota student performance showed a weakness on subobjectives related to interpretation of graphs and tables.

In comparing Minnesota and Central U.S. students on the 51 NAEP items, Minnesota students scored significantly higher on 9 items (18%). There was no significant difference on 32 items (63%), and Minnesota students scored lower on 10 items (20%). The overall average for Minnesota students was 60.4% compared with 59.6% for Central U.S. students.

In analyzing the math subobjectives from which the items were taken, student performance showed strength in using fractions and decimals. There was some weakness evident on subobjectives related to interpretation of graphs and tables.

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[1] The Central United States, as defined by NAEP, consists of Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, South Dakota, and Wisconsin.

TABLE 17
COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE
GRADE 9

SUBOBJECTIVE/DESCRIPTOR		ITEM NUMBER	MN AV. PERCENT CORRECT	NAT'L AV. PERCENT CORRECT	MN/NAT'L PERFORMANCE DIFFERENCE*	CENTRAL U.S. AV. PERCENT CORRECT	MN/CENTRAL U.S PERFORMANCE DIFFERENCE
IB1	Read/Write Numbers	3-18	81.0%	72.9%	H	81.4%	NSD
IB5	Place Value	3-19	76.5%	86.6%	L	86.3%	L
IC1	Geometric Terms	1-19	81.0%	90.7%	NSD	85.6%	L
IC1	Geometric Terms	2-13	69.6%	64.0%	H	72.9%	NSD
IIA1a	Add Whole Numbers	1-5	84.5%	87.3%	L	85.3%	NSD
IIA1a	Add Whole Numbers	2-4	86.9%	88.0%	NSD	90.0%	L
IIA1a	Add Whole Numbers	3-5	81.8%	77.0%	H	80.9%	NSD
IIA1b	Add Decimals	2-5	84.1%	65.4%	H	71.[illegible]%	H
IIA1c	Add Fractions	3-6	69.2%	45.8%	H	46.[illegible]%	H
IIA1c	Add Fractions	4-6	53.8%	39.5%	H	42.9%	H
IIA2c	Multiply Fractions	4-8	56.6%	55.3%	NSD	59.7%	NSD
IIA3a	Division	3-8	79.2%	73.5%	H	76.6%	NSD
IIA3a	Division	4-9	55.1%	51.5%	NSD	53.9%	NSD
IIA5	Decimal/Percent	2-9	52.8%	31.7%	H	30.5%	H
IIA5	Decimal/Percent	4-10	70.5%	75.2%	L	72.6%	NSD
IIA7	Order Decimals	3-22	67.3%	54.0%	H	62.7%	NSD
IIA13	Find Percent	1-12	10.0%	7.7%	H	9.2%	NSD
IIA13	Find Percent	4-11	19.4%	10.5%	H	11.2%	H
IIA14	Rename Fractions	3-10	88.9%	87.2%	NSD	91.1%	NSD
IIA14	Rename Fractions	3-11	75.5%	66.4%	H	69.4%	H
IIA14	Rename Fractions	3-12	75.0%	68.4%	H	75.3%	NSD
IIA18	Order Fractions	1-11	5.6%	2.3%	H	3.7%	NSD
IIC1	Measurements	3-24	90.8%	91.0%	NSD	92.0%	NSD
IIC1	Measurements	4-12	73.9%	60.6%	H	72.9%	NSD
IIC2	Area/Volume/Perimeter	1-13	73.4%	80.6%	L	83.1%	L
IIC2	Area/Volume/Perimeter	1-14	61.0%	62.3%	NSD	68.4%	L
IIC2	Area/Volume/Perimeter	3-13	35.5%	26.2%	H	29.1%	H
IIC2	Area/Volume/Perimeter	4-22	74.3%	69.0%	H	74.4%	NSD
IID5	Interpret Graphs	2-28	56.4%	64.3%	L	68.5%	L
IID5	Interpret Graphs	4-23	88.4%	85.8%	NSD	90.9%	NSD
IID5	Interpret Graphs	4-24	91.3%	93.0%	NSD	97.2%	L
IID5	Interpret Graphs	4-25	36.5%	37.9%	NSD	42.6%	L
IIIA1	Algebra-Expression	4-26	88.5%	86.8%	NSD	88.8%	NSD
IIIB1	Coordinate Plane	2-11	33.0%	24.6%	H	29.2%	NSD
IVA1	Solve Verbal Problem	1-29	95.8%	94.7%	NSD	95.6%	NSD
IVA1	Solve Verbal Problem	2-12	36.3%	25.6%	H	31.0%	NSD
IVA1	Solve Verbal Problem	2-13	95.4%	97.3%	L	96.9%	NSD
IVA1	Solve Verbal Problem	4-15	34.4%	28.1%	H	32.4%	NSD
IVA1	Solve Verbal Problem	4-27	17.5%	20.2%	L	21.2%	NSD
IVA3	Insufficient Data	3-15	53.9%	24.7%	H	27.6%	H
IVA3	Insufficient Data	3-20	66.1%	60.4%	H	66.7%	NSD
IVA6	Estimate Results	1-31	31.8%	26.0%	H	33.9%	NSD
IVA6	Estimate Results	2-30	57.7%	50.4%	H	55.9%	NSD
IVC4	Area/Perimeter	2-31	59.1%	52.9%	H	56.3%	NSD
IVC4	Area/Perimeter	3-16	9.6%	4.8%	H	4.3%	H
IVE1	Reading Tables	3-31	17.8%	21.8%	L	25.1%	L
IVE1	Reading Tables	3-32	18.5%	23.1%	L	26.6%	L
IVE1	Reading Tables	3-33	84.1%	83.0%	NSD	84.6%	NSD
VA3	3-D Drawing	4-34	78.3%	76.9%	NSD	82.5%	NSD
VB2	Discount on Sale	4-35	22.9%	21.1%	NSD	21.0%	NSD
VD	Solving Problems	4-36	74.5%	76.8%	NSD	79.2%	NSD
Overall Average			60.4%	56.1%		59.6%	

Key: H - Minnesota significantly higher than comparison group performance.
L - Minnesota significantly lower than comparison group performance.
NSD - No statistically significant difference between scores.

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*Computed at .05 level of significance.

Conclusions

From the comparisons with National and Central U.S. groups of students the following conclusions seem justified:

1. Minnesota students generally outperformed their National counterparts, while their performance was substantially equal to students in the Central U.S.
2. Minnesota students showed strength on items related to math subobjectives in computation, reasoning, using fractions and decimals, and generalization in geometry. Even though Minnesota students showed strength in geometry when compared to national students, their performance was considered by Minnesota subject specialists to be below an acceptable level.
3. Minnesota students showed a weakness on items related to math subobjectives in interpreting graphs and tables and problem solving. This should be a matter of concern as developmental activities in mathematics programs are planned and carried forward.

STUDENT PERFORMANCE IN CATEGORIES OF DISTRICTS

In the assessment, the total sample of eighth grade students in Minnesota was selected proportionally from the following categories of school districts:

Category 1: Cities of the First Class - Duluth, Minneapolis, St. Paul.

Category 2: Seven-County Metropolitan Area (Developmental Region 11), excluding districts in Category 1.

Districts not included in Categories 1 and 2 with the following K-12 enrollments:

Category 3: 2,000 and Over Students.

Category 4: 1,000-1,999 Students.

Category 5: 1-999 Students.

Table 18 summarizes the average performance of Minnesota students on the mathematics assessment by school district category. A more complete table is contained in the Appendix (Section B3, page 119).

TABLE 18
STUDENT PERFORMANCE BY SCHOOL DISTRICT CATEGORY

All Minnesota Students	1 Cities of 1st Class	2 7-county Metro Area	3 2,000+ Students	4 1,000-1,999	5 1-999 Students
65.4%	55.5%	68.8%	67.1%	63.2%	65.3%

Summary

An analysis of the data in Table 18 reveals that students in district categories 2 (7-county Metropolitan Area) and 3 (2,000 and Over Students) enjoyed a slight advantage when compared to the state average. Students in district categories 4 (1,000-1,999 Students) and 5 (1-999 Students) tended to score close to the state average, but slightly below, while students in district category 1 (Cities of the 1st Class) scored below the Minnesota average by almost ten points.

The consistently lower performance by students in one category of school districts is a cause for concern, and additional research should focus on this characteristic and possible solutions.

THE RELATIONSHIP BETWEEN REPORTING VARIABLES AND MATHEMATICS PERFORMANCE

In a student information section in the assessment, participants were asked to respond to questions relating to their perceptions, attitudes, and background. Responses were then cross tabulated with overall performance for analysis. For example, data reveal that students who reported studying the metric system had an average total test performance of 67.0%, compared to 59.3% for those who reported they had not studied metrics. While the entire student information section is displayed in the Appendix (Section B4, page 120), only selected student responses were chosen for discussion purposes. Selected variables and performance data are summarized in Table 19.

TABLE 19
COMPARISONS OF SELECTED REPORTING VARIABLES WITH OVERALL PERFORMANCE
GRADE 8 *

REPORTING VARIABLE	OVERALL PERFORMANCE
Sex:	
Boys	64.4%
Girls	64.4%
Use of Calculators:	
Not at all	64.7%
Once in awhile	66.2%
Everything except tests	65.6%
Everything	61.1%
Study Metric System:	
No	59.3%
Yes	67.0%
Use of Manipulatives:	
None	53.9%
1 - 6	60.9%
7 - 9	67.2%
10 - 15	71.7%
Class Material:	
One textbook	66.7%
More than one text	64.7%
Individual worksheets	61.2%
Self Appraisal:	
Bottom	46.4%
Well below average	52.1%
Slightly below average	53.0%
Average	61.2%
Slightly above average	71.8%
Well above average	77.2%
Feelings About Math:	
Do not like	54.5%
Least favorite	57.3%
Not among favorites	65.4%
Among favorites	68.8%
Most favorite	68.5%

*Responses reflect those students providing requested information.

Summary

From the data on the student information section of the assessment, the following conclusions seem justified:

1. There was no significant difference between the performance of boys and girls on the assessment.

2. Attitudes toward self and attitudes toward mathematics were positively correlated with results. Those students who felt they were above average in relation to other students scored higher than students reporting themselves average or below. Also, students who reported liking mathematics performed higher than students reporting not liking mathematics.

3. Four factors were consistently related to successful performance. None of the factors are unusual elements in an eighth grade math program. The factors included continued or increased opportunities to learn the metric system, use of a calculator for some activities, use of many manipulative aids, and focusing class work on textbooks.

4. A majority of students (55%) reported that they were either not allowed or not given the opportunity to use a calculator in the classroom. At the eighth grade, opportunities should be presented for students to work with calculators, since other data noted above supports the use of calculators to some extent. Also, 17% of the students reported they had not studied the metric system. The metric system should be included in the curriculum before the eighth grade as well as continued in the eighth grade.

Considering the reporting variables which have an impact on student performance, it is possible to generate a composite picture of a participant with the potential of high performance on a majority of the mathematics objectives. It is also possible to construct a composite picture of a participant with the potential of low performance on a majority of the objectives.

A participant with the potential for high performance on a majority of the objectives:

-Lives in a suburban school district;
-Has an above average self-appraisal;
-Plans to complete college work beyond the baccalaureate level;
-Has an above average self rating in math ability;
-Includes mathematics as one of the favorite subjects;
-Uses a calculator once in awhile;
-Has studied the metric system;
-Uses many (10-15) manipulative aids; and
-Studies mathematics from textbooks.

A participant with the potential for low performance on a majority of the objectives:

- -Lives in a large city school district;
- -Has a very low self-appraisal;
- -Does not plan to finish high school;
- -Has a very low self rating in math ability;
- -Does not like mathematics;
- -Uses a calculator for everything;
- -Has not studied the metric system;
- -Does not use any manipulative aids; and
- -Studies mathematics from a set of worksheets.

The implications of the above composite pictures are complex and far reaching. Some of the issues can be relatively easily addressed by educators while others are much more complex and reach into the social fabric of our society. Besides complexity, the issues also have an importance and a relevance for those concerned about the aims of education and the future of students. As a result of the assessment data, the issues should be studied with the idea of seeking answers and/or solutions to the problems posed.

MINNESOTA 1974 AND 1978 PERFORMANCE COMPARISONS

This section discusses meaningful comparisons which can be made between mathematics performance in 1974 and 1978. In those instances where changes in program procedures limit direct comparability between 1974 and 1978 performance, the nature of limitations in comparability will be noted. The discussion will focus upon the following four comparisons: 1) student performance on identical items; 2) student performance relative to national groups; 3) student performance of boys and girls; and 4) student performance on identical item clusters.

Performance on Identical Items

In 1974, student testing was conducted with thirteen-year-old students. In 1978, student testing was conducted with eighth graders. In order to provide comparability 1974 data were re-analyzed to obtain results from eighth grade students only. All other testing procedures were compatible between the years and require no explanation.

Forty-one of the items in the 1978 mathematics assessment were identical with those administered in 1974. A summary of eighth grade performance on these items is provided in Table 20. As can be seen in the table, the 1978 group significantly outperformed the 1974 group on five items (12%). There was no significant difference in scores between the two groups on 24 items (59%), and on 12 items (29%), the 1978 group performed below the 1974 group. The average performance on all items for the 1978 group was 70.9%, and the average performance for the 1974 group was 72.2%. In summary, the performance of the 1978 group was below that of the 1974 group on seven items more than it was above. However, it should be noted that the decline in average performance across all 41 items was very slight (1.3%) and is not given to represent an educationally meaningful difference.

It is encouraging to note that student performance increased from 1974 to 1978 on items related to computation with fractions. This area has received considerably greater curricular emphasis between testing periods than prior to 1974. This increased emphasis can be viewed to have generated from the 1974 assessment data. However, the slight overall decrease in performance should provide a focus for further curriculum analysis and developmental activities for mathematics education.

Finally, it has been speculated, in conjunction with National Assessment reporting, that a "back to basics" emphasis may, in fact, result in general declines in student performance in mathematics. This phenomenon is certainly suggested by the current data. Eleven of the twelve items for which 1978 performance was significantly lower than in 1974 were related to non-computational areas. Of the five items on which 1978 performance

TABLE 20
1974 and 1978 PERFORMANCE COMPARISONS

CLUSTER/DESCRIPTOR	PACKAGE ITEM	1978 PERF.	1974 PERF.	1978 vs. 1974 PERFORMANCE*
B1 Basic Facts of Arithmetic	2-3	94.3%	98.1%	L
B1 Basic Facts of Arithmetic	3-2	95.0%	93.8%	NSD
B1 Basic Facts of Arithmetic	3-3	97.8%	97.8%	NSD
B1 Basic Facts of Arithmetic	4-1	97.2%	94.4%	H
C1 Computation: Whole Numbers	4-4	85.9%	85.1%	NSD
C1 Computation: Whole Numbers	2-6	88.4%	90.3%	NSD
C1 Computation: Whole Numbers	3-7	79.0%	80.7%	NSD
C2 Computation: Fractions	3-38	76.1%	67.5%	H
C2 Computation: Fractions	3-39	69.4%	70.8%	NSD
C2 Computation: Fractions	3-40	70.2%	69.4%	NSD
C3 Computation: Decimals	1-6	52.6%	52.2%	NSD
C3 Computation: Decimals	4-5	66.4%	57.5%	H
C3 Computation: Decimals	4-7	68.1%	61.7%	NSD
C3 Computation: Decimals	1-10	50.4%	52.4%	NSD
D1 Terms and Symbols	4-17	81.8%	83.6%	NSD
A1 Algebraic Expressions	2-20	76.8%	83.8%	L
A1 Algebraic Expressions	1-23	58.6%	67.0%	L
A1 Algebraic Expressions	3-23	72.8%	79.0%	L
A2 Algebraic Applications	2-10	44.2%	52.1%	NSD
A2 Algebraic Applications	2-26	21.4%	20.7%	NSD
A2 Algebraic Applications	3-27	24.7%	24.9%	NSD
G2 Applications: Geometry	4-30	60.2%	60.7%	NSD
I1 Graphs, Maps, Tables	1-26	99.4%	99.3%	NSD
I1 Graphs, Maps, Tables	1-27	93.8%	96.1%	L
I1 Graphs, Maps, Tables	1-28	99.1%	99.7%	NSD
I1 Graphs, Maps, Tables	2-33	65.6%	72.5%	L
I1 Graphs, Maps, Tables	1-38	52.3%	62.6%	L
M1 Measurement Systems	1-15	90.5%	89.3%	NSD
M1 Measurement Systems	1-16	73.3%	85.2%	L
MM1 Metric Measurement	4-19	94.3%	88.7%	H
MM1 Metric Measurement	1-25	22.7%	24.9%	NSD
P1 Properties of Numbers	2-22	86.9%	92.0%	L
P1 Properties of Numbers	2-23	76.5%	71.7%	NSD
P1 Properties of Numbers	2-24	66.1%	60.0%	NSD
PS1 Basic Problem Solving	3-14	81.8%	83.5%	NSD
PS2 Advanced Problem Solving	3-28	79.7%	83.2%	NSD
PS2 Advanced Problem Solving	1-17	73.7%	85.7%	L
PS2 Advanced Problem Solving	2-15	56.0%	63.0%	L
PS2 Advanced Problem Solving	1-37	24.5%	28.4%	NSD
PS2 Advanced Problem Solving	2-16	72.0%	81.5%	L
PS2 Advanced Problem Solving	3-36	65.5%	50.1%	H
Overall Performance		70.9%	72.2%	

Key:
H - Minnesota 1978 significantly higher than Minnesota 1974 group performance.
L - Minnesota 1978 significantly lower than Minnesota 1974 group performance.
NSD - No statistically significant difference between group scores.

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*Computed at .05 level of significance.

was superior to 1974 performance, three were in areas of computation. It is not possible to attribute definite causality in this regard, but a suggestive performance trend is clearly apparent and should receive continued attention.

National Comparisons

The fact that national data for eighth graders is not available from 1974 limits the extent to which direct statistical comparisons can be made. However, it is possible to make reasonably valid observations where the data for each group are presented separately and where the characteristics of the groups are taken into account. Table 21 summarizes these basic data.

When one observes the number of items on which Minnesota students were statistically above and below national groups, it is apparent that Minnesota thirteen-year-olds in 1974 outperformed their National and Central U.S. counterparts to a greater extent than eighth graders in 1978. Of special interest is the fact that in 1974 Minnesota thirteen-year-olds were below the National and Central U.S. groups on only one item, while in 1978, Minnesota eighth graders were below National and Central U.S. groups on nine and ten items respectively. In 1974, Minnesota thirteen-year-olds outperformed the nation on NAEP items by 5.5% and the Central U.S. by 2.6%. In 1978, Minnesota eighth graders outperformed the nation by 4.3% and the Central U.S. by 0.8%.

As indicated previously, direct statistical comparisons are not possible, but the data can be appropriately interpreted to suggest that Minnesota eighth graders in 1978 did not outperform their eighth grade national counterparts to the extent to which Minnesota thirteen-year-olds in 1974 outperformed its comparison group.

It seems appropriate to suggest that subject matter specialists undertake subsequent analyses to identify meaningful trends across these data which might have implications for curricular emphasis.

TABLE 21

1974 and 1978 NATIONAL COMPARISONS

	1974 (Thirteen-Year-Olds)					1978 (Eighth Graders)				
	NUMBER OF ITEMS	MN. ABOVE	MN. NO DIFFERENCE	MN. BELOW	AVERAGE PER-FORMANCE	NUMBER OF ITEMS	MN. ABOVE	MN. NO DIFFERENCE	MN. BELOW	AVERAGE PER-FORMANCE
National	54	33	20	1	48.1%	51	26	16	9	56.1%
Central U.S.	54	11	42	1	51.0%	51	9	32	10	59.6%
Minnesota	54	--	--	--	53.6%	51	--	--	--	60.4%

Performance of Boys and Girls

In 1974, the average performance on the test for thirteen-year-old girls was 57.2%, while the performance for boys was 56.3%. In 1978, the average performance for both eighth grade boys and girls was 64.4%. There appears to be no relative advantage or meaningful trends in these data. Boys and girls are performing at about the same relative level.

Cluster Analysis

Fourteen of the seventeen clusters for which performance was given earlier in this report were identical with those reported in 1974. It should be clear that items within clusters changed, and that the incompatibility between thirteen-year-olds and eighth graders exists in these data. However, meaningful comparisons can be made between professional judgments across time due to the fact that appropriate criteria were established in conjunction with each of the 1974 and 1978 assessments. It should be apparent that direct statistical comparisons are not meaningful. Data relevant to this discussion are presented in Table 22.

It is apparent from the table that professional expectations as well as student performance on these clusters were both higher in 1978 than in 1974. It is possible that this situation might be attributed to item difficulty levels or to the fact that in 1974 approximately one-third of the students tested were seventh graders. In either case, these factors were taken into account in establishing criteria, and scores tend to vary in a relatively systematic fashion.

The clusters reflecting *Computation With Whole Numbers* (C1), *Maps, Graphs, and Tables* (I1), and *Advanced Problem Solving* (PS2) were strengths in 1974 as well as in 1978. The only area of need across the years was in *Geometric Applications* (G2).

TABLE 22

1974 and 1978 CLUSTER COMPARISONS

	1974 (THIRTEEN-YEAR-OLDS)			1978 (EIGHTH GRADERS)		
	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	JUDGMENT*	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	JUDGMENT *
C1 Computations/ Whole Numbers	70%	87%	STRENGTH	86%	90%	STRENGTH
C2 Computations/ Fractions	54%	39%	NEED	61%	63%	ACCEPTABLE
C3 Computation/ Decimals	53%	37%	NEED	54%	53%	ACCEPTABLE
P1 Properties of Numbers	56%	55%	ACCEPTABLE	79%	76%	NEED
D1 Terms and Symbols	60%	56%	STRENGTH	75%	70%	ACCEPTABLE
G1 Recognition/ Geometry	60%	69%	STRENGTH	60%	55%	NEED
G2 Geometric Applications	65%	43%	NEED	69%	58%	NEED
A1 Algebraic Expressions	60%	50%	NEED	65%	65%	ACCEPTABLE
A2 Algebraic Applications	42%	38%	NEED	51%	51%	ACCEPTABLE
M1 Measurement Systems	50%	56%	STRENGTH	86%	89%	ACCEPTABLE
T1 Graphs, Maps, Tables	60%	70%	STRENGTH	67%	70%	STRENGTH
PS1 Basic Problem Solving	40%	57%	STRENGTH	69%	60%	ACCEPTABLE
PS2 Advanced Problem Solving	41%	50%	STRENGTH	47%	51%	STRENGTH
MM1 Metric Measurement	47%	59%	STRENGTH	74%	63%	NEED

*Judgments for 1978 have been collapsed into three categories as was done for reporting in 1974.

Those areas in which judgments of performance against criteria were more positive in 1978 than in 1974 were: *Computation With Fractions* (C2), *Computation With Decimals* (C3), *Algebraic Expressions* (A1), and *Algebraic Applications* (A2). It is interesting to note that each of these areas was classified as a need area in 1974. It appears that awareness and dissemination activities in these areas may, at least in part, be responsible for these positive changes.

Five areas received generally lower ratings in 1978 as compared with 1974. These areas are as follows: *Properties of Numbers* (P1), *Terms and Symbols* (D1), *Recognition/Geometry* (G1), *Basic Problem Solving* (PS1), and *Metric Measurement* (MM1). Of these five clusters, only P1, G1, and MM1 now represent needs. In the case of metric measurement, this situation probably reflects very high expectations in this area.

As was noted in the discussion of identical items earlier, there appears to be considerable strength in the clusters dealing with computational skills. Each of the three clusters which overlapped from 1974 to 1978 either remained a strength or was removed from the need category. The only computational area of concern in 1978 was *Computation With Integers* (C4), which did not exist as a separate cluster in 1974. It is also worth noting that the cluster *Basic Facts of Arithmetic* (B1), which was not present in 1974, was rated as a strength in 1978. No other systematic patterns appear to emerge from the data.

Conclusions: 1974 and 1978 Comparisons

Based upon a careful review of the data and in line with the previous discussion, the following points seem justified:

1. The weight of the evidence seems to indicate that the performance of Minnesota students shows a slight overall decline which is not considered to be educationally meaningful.

2. Minnesota students, by all indicators are generally performing well in areas of basic arithmetic and computational skills. It is suggested from the data that careful consideration be given to the current strength of Minnesota's performance in these basic areas so that any additional emphasis here is sufficiently focused as not to detract from mastery of higher order skills and processes.

3. The performance of boys and girls was virtually identical in 1978, and no trends are apparent based upon 1974 data.

4. Results of cluster analysis comparisons are viewed as generally positive in that it is possible to show that most major needs identified in 1974 were rectified. This situation can, at least in part, be attributed to awareness and positive action on the part of mathematics educators and other decision makers.

5. Although results against criteria showed considerable overall strength, some needs clearly exist. Positive strategies, based upon resultant recommendations, should be carefully considered, documented, and acted upon where appropriate.

RECOMMENDATIONS FOR IMPROVING MATHEMATICS INSTRUCTION

The following recommendations are suggested by the data, but to some extent have been supplemented by additional observations by math educators.

A. Teacher Education

1. Inservice

a. Programs on the appropriate use of the calculator within mathematics classrooms should be developed. Staff development programs in the judicious use of the calculator in seventh and eighth grade mathematics must be provided, along with the necessary curriculum revisions.

b. The importance and development of estimation skills should be presented in a staff development program.

c. Concepts that pertain to the application of mathematics in daily living (percent, decimals in computation, geometric and algebraic skills) need to be highlighted in the junior high mathematics curriculum. It may be necessary for mathematics teachers to develop units or projects which emphasize application of basic computational skills.

2. Preservice

a. College mathematics courses required for the preparation of secondary mathematics teachers should include elements of the following:

1) Application skills

        2) Metric measurement; and

        3) Contemporary notation for large numbers.

    b. College methods courses required for the preparation of secondary mathematics teachers should include elements of the following:

        1) Appropriate use of the calculator and manipulative materials in the mathematics classrooms;

        2) Methods of teaching the skill of estimation; and

        3) Methods of teaching computational algorithms for decimals as well as fractions.

B. Curriculum

1. Textbooks

    a. A single textbook seems to be associated with highest performance and should probably be the focus for the junior high school mathematics curriculum.

    b. Selection of a textbook series must take into consideration the presentation of at least the following:

        1) Appropriate exercises requiring the use of a calculator;

        2) Opportunities to interpret charts and graph data; and

        3) Inclusion of informal geometry, especially solid geometry such as determining top or side views of solids and of interpreting 2-dimensional drawings of 3-dimensional solids.

2. Support or supplementary material

    a. Within the mathematics curriculum, opportunities should be presented which fostor positive attitudes toward mathematics.

    b. Positive models of career roles and professional opportunities should be presented in order to increase the desire to pursue mathematics courses in future schooling.

3. Technology

    a. Calculators should be available for appropriate learner use.

    b. Manipulative aids should be an integral part of the mathematics curriculum. In the area of geometry, physical models of plane and solid objects should be available for learner use.

C. School/District-wide Policy

1. Mathematics curriculum objectives should be formulated which include statements concerning where, when, and how the metric system will be taught.
2. Mathematics curriculum objectives should be formulated which include statements concerning the appropriate use of the calculator within the mathematics classroom.
3. Financial support should be provided to purchase manipulative aids.
4. The selection of a single textbook series as the focus of the mathematics curriculum should be considered on a system-wide basis.
5. The teaching strategies required for application skills should be developed on a system-wide basis.

D. Minnesota State Education Policy

1. Leadership in the appropriate role of the calculator in the junior high mathematics curriculum should be exhibited by helping school systems develop effective calculator use policies.
2. Suggested manipulative aids and manipulative aids guidelines for different aspects of the mathematics curriculum should be developed and disseminated. Such guidelines would assist individual school systems to make appropriate decisons for the purchase and use of manipulative aids.
3. As was done with the concept of fractions in the mathematics curriculum a few years ago, a position paper should be formulated for teaching the concept of decimals and percents in the mathematics curriculum.

ANALYSIS OF ERRORS ON MATHEMATICS EXERCISES

An analysis of errors on the eighth grade mathematics assessment is contained in the Appendix (Section B5, page 122). From the analysis, the following conclusions seem justified:

1. Multi-digit divisor problems caused some difficulty for eighth grade participants.
2. Successful computation with fractions has clearly increased since 1974. Typical error responses have also decreased over the four year interval.
3. Estimation skills should be emphasized with decimals. Also, place value and decimal point placement difficulties seemed to occur frequently.
4. Contemporary notation for writing large numbers with a space rather than commas should probably receive greater emphasis in the eighth grade.

5. The symbol ≠ should be taught in the seventh and eighth grade mathematics curriculum.

6. Geometric application such as top views of solids or the number of cubes in a solid were not familiar to many eighth grade students.

7. Severe difficulties by eighth grade students in reading and interpreting chart headings of titles was evident.

8. Metric measurement awareness and knowledge has definitely increased in the interval from 1974 to 1978.

9. The context of a word problem seemed to be a significant factor in the success of solving the problem. A menu context problem had a high success rate while a savings account problem had a very low rate of success.

10. Application of math skills outside the classroom probably should receive greater emphasis.

Summary

Analysis of errors on mathematics exercises by eighth grade students showed typical error patterns in computational skills and a definite need to increase application skills and certain problem solving skills. Students need skill development in multi-digit divisor problems as well as other areas identified. These areas should provide a focus for continued developmental activities in junior high mathematics.

CHAPTER 5

ELEVENTH GRADE ASSESSMENT

CHAPTER 5

ELEVENTH GRADE ASSESSMENT

INTRODUCTION

The eleventh grade mathematics assessment was conducted in April, 1979. A total of 9,800 students, or about 13% of the eleventh grade public school population was included in the assessment. Students were selected by means of a random sampling procedure from five categories of school districts. The size of the sample and the random nature of the selection means that generalizations from the data can be made to the total population of eleventh graders with a high degree of confidence.

PROCEDURES

Instrument Development

The instruments in the Minnesota Educational Assessment Program are criterion-referenced. That is, questions are developed to measure student performance relative to specified goals, objectives, and subobjectives. The primary source for item development is the publication, Mathematics Objectives, Eleventh Grade (1978) which was developed in the Division of Instruction (See Appendix, Section C4, page 152). Results are essentially reported against these goals, objectives, and subobjectives.

For the mathematics assessment, the following five major goal areas were addressed across grade levels:

1. RECALLING AND/OR RECOGNIZING DEFINITIONS, FACTS, AND SYMBOLS.

   A task assessing this ability requires that the student be able to recognize typical mathematical symbolism, or to recall specific facts.

2. PERFORMING MATHEMATICAL MANIPULATIONS.

   This includes tasks which require the student to carry out single operations and procedures (or sequences of these) that have been previously learned and are specifically requested.

3. UNDERSTANDING MATHEMATICAL CONCEPTS AND PROCESSES.

   This level includes tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another.

4. SOLVING MATHEMATICAL PROBLEMS--SOCIAL, TECHNICAL, AND ACADEMIC.

   Assessment of the ability to solve problems requires the student to demonstrate the ability to select knowledge, skills, information, and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

5. USING MATHEMATICS AND MATHEMATICAL REASONING TO ANALYZE PROBLEM SITUATIONS, DEFINE PROBLEMS, FORMULATE HYPOTHESES, MAKE DECISIONS, AND VERIFY RESULTS.

   This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Because several subobjectives and individual exercises fall into categories considered consistent with common curricular designations, an additional clustering of exercises was completed by a committee of mathematics educators for purposes of analysis. Each of these clusters incorporates several subobjectives and provides a sufficient number of exercises to allow maximum generalizability of results. Eighteen clusters were identified for eleventh grade students.

In addition, of the 192 items in the assessment, 57 (30%) were selected from the National Assessment of Educational Progress (NAEP) in order to provide comparisons with national groups of students. The remaining items were developed by subject specialists working with Department of Education personnel.

REPORTING OF DATA

In the following sections, data from the assessment are reported in a variety of formats in order to provide as comprehensive an analysis as possible of Minnesota student performance in mathematics.

PERFORMANCE BY SUBOBJECTIVES AND CLUSTERS

The evaluation of student performance on subobjectives is displayed in Table 23. Because of the large number of mathematics subobjectives, Table 23 displays a summary of student performance. A more complete table is contained in the Appendix (Section C1, page 145).

TABLE 23
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 11

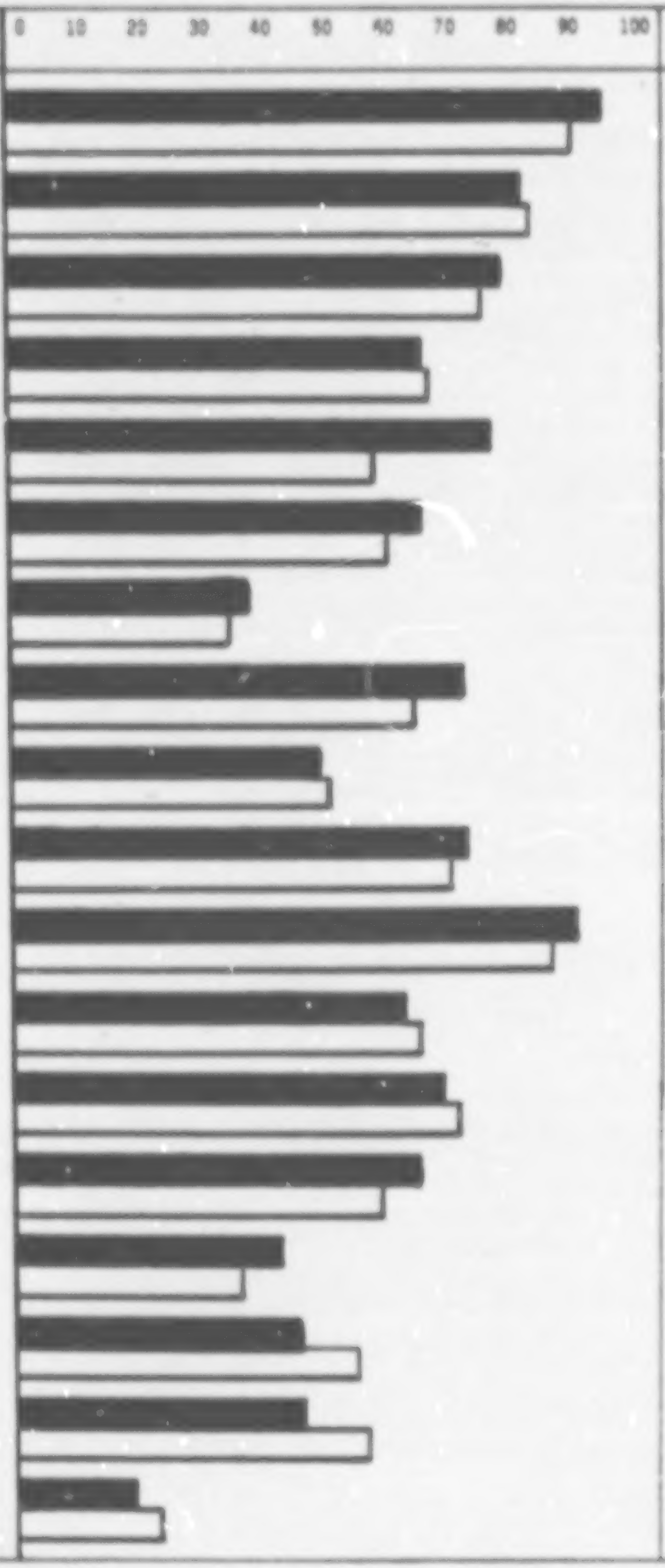
EVALUATION	SUBOBJECTIVES
Strength	IA1, IB1, IC3, ID4, IE2, IE6, IJ1, IIA10, IIC2, IID5, IIE1, IIE3, IIIB1, IIIB3, IVD1, IVE2, VB1, VC4
Potential Strength	ID1, IIA1, IIA7, IIA9, IID6, IIIA2, IIIB4, IVA5, VA1, VB2, VD1
Acceptable	IC8, ID2, II1, II2, IIA2, IIA3, IIB7, IIC1, IID1, IID4, IVD4, IVE1, IVG2
Potential Need	IC4, IC7, IIA6, IIA8, IIB2, IIB11, IIB12, IIIB2, IVA1, IVD2, IVG1, VB3
Need	IB3, IIB3, IIE2, IVB1, IVB2, IVG4, IVH1, VB4, VC2

Student performance on clusters relative to criteria established by subject specialists is displayed in Table 24. The definition and general content of the clusters is contained in the Appendix, Section C2, page 147.

Criteria for acceptable student performance on clusters were established by subject specialists in the same manner as criteria for student performance on subobjectives, described earlier. Different committees of educators were used for the two criteria setting processes, thereby allowing different groups to judge assessment results. The same five point scale utilized in evaluating performance on subobjectives was applied (strength, potential strength, acceptable, potential need, need).

TABLE 24
STUDENT PERFORMANCE ON MATHEMATICS CLUSTERS
GRADE 11

Cluster/Descriptor	Actual Perf. Criteria*	Evaluation*
B1 - Basic Facts of Arithmetic	96% 90%*	Strength
C1 - Computation With Whole Numbers	82% 83%*	Acceptable
C2 - Computation With Fractions	79% 74%*	Potential Strength
C3 - Computation With Decimals	66% 67%*	Acceptable
C4 - Computation With Integers	77% 59%*	Strength
A1 - Algebraic Expressions	66% 60%*	Potential Strength
A2 - Algebraic Applications	37% 33%*	Acceptable
G1 - Geometric Properties	72% 65%*	Potential Strength
G2 - Applications of Geometry	49% 50%*	Acceptable
I1 - Interpret Graphs, Etc.	73% 71%*	Acceptable
M1 - Measurement Systems	93% 87%*	Strength
MM1 - Metric Measurement	64% 66%*	Acceptable
P1 - Properties of Numbers	70% 73%*	Potential Need
PS1 - Basic Problem Solving	65% 60%*	Potential Strength
PS2 - Patterns, Logic, Advanced Problem Solving	43% 33%*	Potential Strength
PS3 - Consumer Problems	47% 56%*	Need
SP1 - Probability and Statistics	50% 59%*	Need
T1 - Trigonometry	18% 21%*	Potential Need



*Criteria and the evaluation of student performance were determined by subject specialists.

The dark bars/light bars represent actual performance/expectations.

Summary of Performance on Subobjectives and Clusters

The Roman numeral preceeding each subobjective (Table 23) identifies the goal to which it is assigned. The five goals reflect an ascending order of cognitive difficulty, with Roman I representing the lowest level (recall, recognition) of cognitive classification and Roman V (reasoning, analysis) representing the highest.

An analysis of student performance on subobjectives (Table 23) shows that students generally performed better on lower level cognitive goals related to recall, recognition, and computation, and poorer on higher level cognitive goals related to application, problem solving, and decision making. These findings relative to Minnesota students are consistent with the results from the National Assessment of Educational Progress (NAEP) and indicates that this trend in student performance is nationwide in scope.

Similar findings are apparent when performance is analyzed by cluster (Table 24). Different groups of subject specialists established criteria for performance, and while classification levels may differ somewhat, relative performance on subobjectives and clusters is fairly consistent.

According to the judgments of subject specialists, student performance showed strength in basic facts of arithmetic, computation with fractions and integers, basic ideas of algebra and geometry, knowledge of measurement systems, and advanced as well as basic problem solving, patterns and logic.

Student performance was determined to be acceptable on clusters dealing with computation with whole numbers and decimals, application of algebra and geometry, interpreting graphs, maps, and tables, and metric measurement.

Clusters where performance was determined to be in need of improvement included properties of numbers, consumer problems, and the generally higher levels of secondary mathematics relating to probability and statistics and trigonometry.

Since there is little value in knowing facts or skills without the ability to use them in real situations, much more emphasis needs to be focused on applications and problem solving. It is well established that teaching problem solving and application is not easy. However, with greater emphasis, better examples, more time, and more concrete instruction, higher achievement is likely to occur. The results also suggest more emphasis on probability and statistics, a very important field in today's world.

COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE ON SELECTED ITEMS

In the development of each assessment instrument, an effort is made to include quality items from the National Assessment of Educational Progress (NAEP) if such items also reflect the goals and objectives of Minnesota educators. The inclusion of NAEP items thus allows for comparisons with national and Central United States[1] groups of students.

Of the 192 knowledge and skills items in the assessment, 57 (30%) were from NAEP. Minnesota and national performance comparisons are displayed in Table 25.

Summary

On the 57 items on which comparisons were made, Minnesota students scored above their national counterparts on 28 items (49%). On 20 items (35%) there was no significant difference, and on 9 items (16%), Minnesota students scored lower than the national group. The overall average for Minnesota students on the 57 items was 63.6% compared with 59.8% for the national group.

In analyzing math subobjectives from which the items were taken, Minnesota students showed strength on items related to adding and dividing whole numbers, percentage problems, number equivalencies, and area. Minnesota student performance showed some weakness on items related to terms and figures.

In comparing Minnesota and Central U.S. student performance on the 57 items, Minnesota students scored significantly higher on 15 items (26%). There was no significant difference on 30 items (53%), and on 12 items (21%) Minnesota students scored significantly lower than their Central U.S.

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[1] The Central United States, as defined by NAEP, consists of Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, and Wisconsin.

TABLE 25
COMPARISON OF MINNESOTA AND NATIONAL PERFORMANCE
GRADE 11

SUBOBJECTIVE/DESCRIPTION	ITEM NUMBER	MN AV. PERCENT CORRECT	NAT'L AV. PERCENT CORRECT	MN/NAT'L PERFORMANCE DIFFERENCE*	CENTRAL U.S. AV. PERCENT CORRECT	MN/CENTRAL U.S. PERFORMANCE DIFFERENCE
IB3 Place Value	1-18	69.9%	80.0%	L	84.4%	L
IC3 Terms-Lines	2-20	79.4%	80.8%	NSD	83.6%	L
IC4 Terms-Figures	1-19	24.8%	57.2%	L	99.7%	L
IC4 Terms-Figures	1-20	40.7%	42.2%	NSD	66.9%	NSD
IC4 Terms-Figures	1-21	26.0%	34.9%	L	36.3%	L
ID2 Identify Yards, Feet, Etc.	4-22	97.0%	96.5%	NSD	95.8%	NSD
ID2 Identify Yards, Feet, Etc.	4-23	87.9%	86.7%	NSD	90.9%	L
ID2 Identify Yards, Feet, Etc.	4-24	94.0%	93.6%	NSD	93.7%	NSD
ID4 Scales in Measurement	3-5	86.7%	84.8%	NSD	91.7%	L
ID4 Scales in Measurement	4-25	97.4%	95.5%	H	96.5%	NSD
IE6 Exponents and Logarithms	2-23	92.1%	94.2%	L	93.3%	NSD
IIA1 Whole Numbers-Add	2-4	89.9%	92.2%	L	92.6%	NSD
IIA1 Whole Numbers-Add	3-1	89.0%	85.0%	H	86.2%	H
IIA1 Whole Numbers-Add	3-2	81.9%	69.0%	H	74.0%	H
IIA1 Whole Numbers-Add	3-3	64.8%	69.5%	L	71.6%	L
IIA1 Whole Numbers-Add	4-2	82.8%	79.2%	H	78.8%	NSD
IIA3 Whole Numbers-Divide	4-4	82.7%	67.8%	H	69.2%	H
IIA3 Whole Numbers-Divide	4-5	60.9%	52.2%	H	54.0%	H
IIA6 Ordering Numbers	1-26	96.4%	95.6%	NSD	97.5%	NSD
IIA6 Ordering Numbers	1-27	78.5%	75.9%	NSD	79.3%	NSD
IIA6 Ordering Numbers	1-28	80.2%	75.6%	H	81.3%	NSD
IIA8 Percentage Problems	1-7	64.5%	56.4%	H	60.6%	NSD
IIA8 Percentage Problems	2-7	50.4%	28.1%	H	32.1%	H
IIA8 Percentage Problems	3-6	26.5%	17.3%	H	19.9%	H
IIA9 Find Equivalency	1-8	90.9%	87.6%	H	89.6%	NSD
IIA9 Find Equivalency	2-8	94.1%	95.0%	NSD	96.7%	L
IIA9 Find Equivalency	2-9	81.2%	82.1%	NSD	83.6%	NSD
IIA9 Find Equivalency	2-10	83.8%	82.7%	NSD	83.8%	NSD
IIA9 Find Equivalency	4-7	71.9%	45.3%	H	50.5%	H
IIA9 Find Equivalency	4-8	85.9%	77.6%	H	80.0%	H
IIA9 Find Equivalency	4-9	86.9%	83.0%	H	86.6%	NSD
IIB3 Quadratic Expressions	4-12	28.1%	23.9%	H	19.3%	H
IIB11 Inequalities	1-30	50.0%	50.6%	NSD	55.9%	NSD
IIB11 Inequalities	2-25	18.9%	20.7%	NSD	20.6%	NSD
IIB11 Inequalities	4-13	40.7%	36.5%	H	38.7%	NSD
IIB12 Factoring	2-12	25.6%	19.3%	H	20.0%	H
IIC1 Geometric Constructions	2-26	45.7%	40.7%	H	47.5%	NSD
IID1 Conversion Relations	3-25	53.7%	53.0%	NSD	54.2%	NSD
IIE3 Probability	3-10	65.0%	37.0%	H	43.5%	H
IVA1 Apply Formulas	1-14	37.3%	20.2%	H	22.9%	H
IVD4 Interpret Table, Chart, Etc.	3-27	93.5%	88.7%	H	93.2%	NSD
IVD4 Interpret Table, Chart, Etc.	3-28	76.3%	69.3%	H	73.3%	NSD
IVD4 Interpret Table, Chart, Etc.	3-29	27.4%	37.7%	L	41.3%	L
IVD4 Interpret Table, Chart, Etc.	4-30	37.6%	34.4%	NSD	35.6%	NSD
IVD4 Interpret Table, Chart, Etc.	4-31	40.2%	35.6%	H	35.3%	NSD
IVD4 Interpret Table, Chart, Etc.	4-32	91.2%	90.4%	NSD	92.5%	NSD
IVS1 Common Household Problems	1-16	39.6%	25.5%	H	25.3%	H
IVS2 Solve Consumer Problems	1-33	61.0%	58.4%	NSD	57.2%	NSD
IVS2 Solve Consumer Problems	1-34	84.9%	89.7%	L	90.9%	L
IVS2 Solve Consumer Problems	1-35	73.4%	72.2%	NSD	73.7%	NSD
IVS2 Solve Consumer Problems	4-35	27.0%	27.0%	NSD	28.8%	NSD
IVS4 Problems Involving Area	3-17	21.2%	13.4%	H	16.9%	H
IVS4 Problems Involving Area	4-17	18.7%	9.9%	H	14.2%	NSD
IVW1 Properties of Numbers	3-33	48.9%	50.7%	NSD	53.6%	L
VB2 Installment Buying	2-18	40.4%	35.0%	H	36.5%	NSD
VB3 Determine Discount	2-30	48.0%	37.2%	H	37.7%	H
VB4 Comparative Buying	4-18	16.0%	19.2%	L	19.3%	L
OVERALL AVERAGE		63.6%	59.3%		62.1%	

Key: H - Minnesota significantly higher than comparison group performance.
L - Minnesota significantly lower than comparison group performance.
NSD - No statistically significant difference between scores.

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*Computed at .05 level of significance.

counterparts. The overall average for Minnesota students on the 57 items was 63.6% compared to 62.1% for Central U.S. students.

In analyzing math subobjectives from which the items were taken, Minnesota students showed strength on items related to adding and dividing whole numbers and on percentage problems.

Minnesota student performance showed some weakness on items related to terms and figures.

Conclusions

From comparisons with national and Central U.S. groups of students, the following conclusions seem justified:

1. Minnesota students substantially outperformed both national and Central U.S. students, although there was a relatively smaller advantage over Central U.S. students.
2. Minnesota students generally showed strength in areas representing basic mathematics skills. These results are encouraging and suggest that Minnesota students have relatively strong mathematics skills.
3. Minnesota students displayed a relative weakness in the area of identifying mathematical terms and figures (IC4). This area was also identified as a potential need by subject specialists who established criteria for acceptable student performance. It appears that this area should be the focus of subsequent developmental activities.

STUDENT PERFORMANCE IN CATEGORIES OF DISTRICTS

In selecting the sample for the assessment, eleventh grade students were selected proportionally from the following categories of school districts:

Category 1: Cities of the First Class - Duluth, Minneapolis, St. Paul.

Category 2: Seven-county Metropolitan Area (Developmental Region 11), excluding districts in Category 1.

Districts not included in Categories 1 and 2 with the following K-12 enrollments:

Category 3: 2,000 and Over Students.

Category 4: 1,000-1,999 Students.

Category 5: 1-999 Students.

The average performance of Minnesota students on the mathematics assessment by school district category is summarized in Table 26.

TABLE 26
STUDENT PERFORMANCE BY SCHOOL DISTRICT CATEGORY
GRADE 11

All Minnesota Students	1 Cities of 1st Class	2 7-county Metro Area	3 2,000+ Students	4 1,000-1,999	5 1-999 Students
64.1%	62.7%	64.6%	65.9%	64.0%	62.5%

Summary

As can be seen from the data in Table 26, students in all five categories of school districts tended to score close to the state average. The district category with the highest average score (2,000 and Over Students) scored 1.8% above the state average, while the district with the lowest average score (1-999 Students) scored 1.6% below the state average. Such consistent performance across the state is a fairly unique phenonomon and seems to indicate that skill and knowledge development across the state at the eleventh grade level is quite uniform.

THE RELATIONSHIP BETWEEN REPORTING VARIABLES AND MATHEMATICS PERFORMANCE

In a student information section of the assessment, participants were asked to respond to questions relating to their perceptions, attitudes, and background. Responses were then cross tabulated with overall performance on the knowledge and skill sections of the assessment to determine if different response patterns were associated with different scores. For example, data reveal that students who reported math to be their least favorite subject had an average total test score of 54.6% compared to an average score of 77.9% for students reporting math to be their favorite subject.

Selected portions of the student information section were chosen for discussion purposes and are displayed in Table 27. The entire student information section is displayed in the Appendix (Section C3, page 149).

Summary

From the data on the student information section of the assessment, the following conclusions seem justified:

1. The overall score for boys on the assessment was 3.3% higher than for girls.
2. Attitudes toward self and attitudes toward mathematics were positively correlated with results. Those students who felt they were smarter than others in their classes or grade scored higher than students who felt they were about as smart or less smart. Students who reported liking math or who felt math was easier than most other subjects scored higher than students reporting not liking math, or who felt that math was harder than most other subjects.
3. Increased use of a calculator in math classes was correlated with higher overall performance.

Considering the reporting variables which have an impact on student performance, it is possible to generate a composite picture of a participant with the potential of high performance on a majority of the mathematics objectives. It is also possible to construct a composite of a participant with the potential of low performance on a majority of the mathematics objectives.

TABLE 27
COMPARISONS OF SELECTED REPORTING VARIABLES WITH OVERALL PERFORMANCE
GRADE 11

REPORTING VARIABLE	OVERALL PERFORMANCE
Sex:	
Boys	66.2%
Girls	62.9%
Self Appraisal:	
Not as smart	49.9%
About as smart	63.2%
Smarter	76.8%
Feelings About Math:	
Least favorite	54.6%
Not among favorites	61.8%
One of favorites	72.8%
Most favorite	77.9%
Compared to Other Subjects:	
Easier	73.1%
About the same	66.1%
Harder	59.7%
Use of Calculators:	
Not at all	58.8%
Once in awhile	65.3%
Everything except tests	66.9%
For everything	69.3%

*Responses reflect those students providing requested information.

A participant with the potential for high performance on a majority of mathematics objectives:

- -Lives in a school district of 2,000 and Over Students;
- -Is a boy;
- -Has an above average self-appraisal;
- -Plans to complete college work beyond the baccalaureate level;
- -Identifies math as a favorite subject;
- -Feels math is easier than most other subjects;
- -Takes mathematics courses in grades 9, 10, and 11; and
- -Uses a calculator.

In contrast, a participant with the potential for low performance on a majority of mathematics objectives:

- -Lives in a school district with less than 1,000 students;
- -Is a girl;
- -Has a low self-appraisal;
- -Plans to leave high school before graduation;
- -Identifies math as a least favorite subject;
- -Feels math is harder than most other subjects;
- -Does not take mathematics in grades 9, 10, or 11; and
- -Does not use a calculator.

While the above composite pictures are helpful in identifying some of the factors associated with mathematics performance, they also suggest a complexity that must be recognized when dealing with issues related to performance. As a result of the assessment, the data should be studied with the aim of establishing directions and priorities for mathematics education in Minnesota.

MINNESOTA 1974 AND 1978 PERFORMANCE COMPARISONS

In 1974, the first Minnesota statewide assessment in mathematics was carried out. This section discusses meaningful comparisons which can be made between mathematics performance in 1974 and 1978. In those instances where changes in program procedures limit direct comparability between 1974 and 1978 performance, the nature of limitations in comparability will be noted. The discussion in this section will focus on the following four comparisons: 1) performance on identical items; 2) performance relative to national groups; 3) performance of boys and girls; and 4) performance on identical cluster groupings.

Performance on Identical Items

In 1974, the assessment was conducted with seventeen-year-old students, while in 1978, testing was conducted with eleventh graders. In order to provide comparability, 1974 data were re-analyzed to obtain results for eleventh grade students. All other testing procedures were compatible between 1974 and 1978 and require no explanation.

Thirty-four items in the 1974 test were included in the 1978 assessment. A comparison of student performance on these items is provided in Table 28.

As can be seen in Table 28, the 1978 group significantly outperformed the 1974 group on one item (3.0%). On 26 items (76.0%) there was no significant difference, and on 7 items (21%), the 1978 group performed significantly below the 1974 group. The average performance on all items was 51.6% for the 1978 group and 52.9% for the 1974 group. This decline in overall performance of 1.3% is slight and is not considered to be an educationally meaningful difference.

While the overall decline in performance between 1974 and 1978 is slight, an analysis of those areas where declines are evident should provide a focus for further analysis and evaluation. Student performance showed a

TABLE 28
1974 AND 1978 ITEM PERFORMANCE COMPARISONS
GRADE 11

CLUSTER/DESCRIPTOR	PACKAGE ITEM	1978 PERF.	1974 PERF.	1978 vs. 1974 PERFORMANCE*
C2 Computation: Fractions	3-3	64.6%	71.4%	NSD
C3 Computation: Decimals	4-1	79.5%	78.0%	NSD
C3 Computation: Decimals	1-5	73.0%	71.6%	NSD
C3 Computation: Decimals	2-6	58.2%	54.7%	NSD
P1 Properties of Numbers	3-32	34.9%	43.1%	L
G1 Geometric Properties	3-20	66.6%	64.6%	NSD
G2 Application of Geometry	1-22	44.7%	37.4%	NSD
G2 Application of Geometry	1-13	76.6%	77.7%	NSD
A1 Algebraic Expressions	4-27	40.7%	44.3%	NSD
A1 Algebraic Expressions	2-11	21.5%	17.4%	NSD
A2 Algebra: Applications	1-9	25.9%	18.8%	NSD
A2 Algebra: Applications	1-11	66.7%	62.2%	NSD
A2 Algebra: Applications	1-12	30.2%	22.9%	NSD
I1 Interpreting Graphs, Etc.	4-14	72.2%	73.0%	NSD
I1 Interpreting Graphs, Etc.	4-34	68.6%	77.6%	L
PS1 Basic Problem Solving	3-14	82.0%	87.3%	L
PS1 Basic Problem Solving	1-36	71.7%	83.5%	L
PS1 Basic Problem Solving	1-37	42.4%	60.8%	L
PS2 Advanced Problem Solving	2-36	81.1%	79.4%	NSD
PS2 Advanced Problem Solving	3-35	75.9%	80.4%	NSD
PS2 Advanced Problem Solving	2-37	25.0%	26.3%	NSD
PS3 Consumer Problem Solving	3-31	53.3%	67.4%	L
PS3 Consumer Problem Solving	3-34	25.4%	37.3%	L
MM1 Metric Measurement	4-21	95.6%	94.9%	NSD
MM1 Metric Measurement	4-47	74.0%	72.7%	NSD
MM1 Metric Measurement	4-48	51.3%	57.3%	NSD
MM1 Metric Measurement	4-49	76.0%	78.5%	NSD
MM1 Metric Measurement	2-35	33.3%	27.6%	H
SP1 Probability and Statistics	1-41	18.6%	14.6%	NSD
SP1 Probability and Statistics	1-42	36.0%	31.5%	NSD
SP1 Probability and Statistics	1-43	22.0%	22.7%	NSD
T1 Trigonometry	1-38	20.5%	19.9%	NSD
T1 Trigonometry	1-39	26.4%	21.2%	NSD
T1 Trigonometry	1-40	20.0%	22.0%	NSD
Overall Performance		51.6%	52.9%	

Key: H - Minnesota 1978 significantly higher than Minnesota 1974 group performance.
L - Minnesota 1978 significantly lower than Minnesota 1974 group performance.
NSD - No statistically significant difference between group scores.

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*Computed at .05 level of significance.

definite decline in the areas of *Basic Problem Solving* (PS1) and *Consumer Problem Solving* (PS3). It is unclear whether there are definite declines in the areas of *Properties of Numbers* (P1) and *Interpreting Graphs, Maps, and Tables* (I1), and these areas will need additional analysis.

Overall, it can be stated that in general there has been relatively little change in mathematics achievement at the eleventh grade level between 1974 and 1978. The two areas where declines were evident should be noted and these areas should receive continued attention.

Performance Relative to National Groups

The fact that national data for eleventh graders from 1974 is not available limits the extent to which direct statistical comparisons can be made. However, it is possible to make reasonably valid observations where the data for each group are presented separately and where the characteristics of the groups are taken into account. Table 29 displays these comparison data.

In 1974, Minnesota seventeen-year-olds outperformed national and Central U.S. students by 3.3% and 1.6% respectively. In 1978, eleventh graders outscored national students by 3.8% and Central U.S. students by 1.5%.

As indicated previously, direct statistical comparisons are not possible, but the data can be appropriately interpreted to suggest that Minnesota eleventh graders in 1978 continue to outperform their national counterparts at least to the extent that Minnesota seventeen-year-olds outperformed the comparison group in 1974, and at least relative to national students, the 1978 Minnesota group seems to have improved their relative advantage.

Performance of Boys and Girls

In 1974, the average performance on the assessment for seventeen-year-old boys was 54.3%, while the performance for girls was 51.8%, a difference

TABLE 29
1974 AND 1978 NATIONAL COMPARISONS
GRADE 11

	1974 Seventeen-Year-Olds					1978 Eleventh Graders				
	NUMBER OF ITEMS	MN ABOVE	MN NO DIFFERENCE	MN BELOW	AVERAGE PERFORMANCE	NUMBER OF ITEMS	MN ABOVE	MN NO DIFFERENCE	MN BELOW	AVERAGE PERFORMANCE
National	37	14	21	2	63.2%	57	28	20	9	59.8%
Central U.S.	37	6	30	1	64.9%	57	15	30	12	62.1%
Minnesota	37	--	--	--	66.5%	57	--	--	--	63.6%

of 2.5%. In 1978, the average performance for eleventh grade boys was 66.2% versus 62.9% for girls, a difference of 3.3%.

The differences in performance for boys and girls from the 1974 and 1978 assessments are slight and are not considered to be educationally meaningful.

Cluster Analysis

Fourteen of the eighteen clusters in the 1978 assessment were also contained in the 1974 assessment. It should be noted that many items within clusters changed, and that incompatibility between seventeen-year-olds and eleventh graders exists in these data. However, meaningful comparisons can be made between professional judgements across time due to the fact that appropriate criteria were established relative to student performance for each of the assessments. It should be apparent that direct statistical comparisons are not meaningful. Thus, the following discussion will focus on comparisons between professional judgements. Data relevant to the discussion are displayed in Table 30.

It is apparent from the table that professional expectations as well as student performance were both higher in 1978 than in 1974. It is possible that this could be attributed to changes in item difficulty levels or to the fact that a number of seventeen-year-olds in 1974 were tenth graders. In either case, these factors were taken into account by subject specialists when establishing criteria.

Student performance on four clusters remained a strength from 1974 to 1978. These included *Geometric Properties* (G1), *Algebraic Expressions* (A1), *Basic Problem Solving* (PS1), and *Advanced Problem Solving* (PS2). Performance relative to subject specialists' judgements declined on three clusters, *Computation With Whole Numbers* (C1), *Properties of Numbers* (F1), and *Algebraic Applications* (A2), although it should be noted that in each instance performance was still considered to be acceptable.

TABLE 30
1974 AND 1978 CLUSTER COMPARISONS
GRADE 11

	1974 SEVENTEEN-YEAR-OLDS			1978 ELEVENTH GRADERS		
CLUSTER/DESCRIPTION	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	JUDGEMENT*	MINIMUM ACCEPTABLE PERCENT	AVERAGE STUDENT PERFORMANCE	JUDGEMENT*
C1 Computation: Whole Numbers	87.0%	92.0%	Strength	83.0%	82.0%	Acceptable
C2 Computation: Fractions	69.0%	61.0%	Need	74.0%	79.0%	Strength
C3 Computation: Decimals	65.0%	52.0%	Need	66.0%	65.0%	Acceptable
P1 Properties of Numbers	55.0%	62.0%	Strength	73.0%	70.0%	Acceptable
G1 Geometric Properties	52.0%	62.0%	Strength	65.0%	72.0%	Strength
G2 Application of Geometry	53.0%	53.0%	Acceptable	50.0%	49.0%	Acceptable
A1 Algebraic Expressions	41.0%	49.0%	Strength	60.0%	66.0%	Strength
A2 Algebraic Applications	30.0%	35.0%	Strength	33.0%	37.0%	Acceptable
M1 Measurement Systems	57.0%	58.0%	Acceptable	87.0%	93.0%	Strength
I1 Interpreting Graphs, Etc.	50.0%	51.0%	Acceptable	71.0%	73.0%	Acceptable
SP1 Probability & Statistics	37.0%	25.0%	Need	59.0%	50.0%	Need
T1 Trigonometry	21.0%	17.0%	Need	21.0%	18.0%	Acceptable
PS1 Basic Problem Solving	59.0%	63.0%	Strength	60.0%	65.0%	Strength
PS2 Advanced Problem Solving	35.0%	45.0%	Strength	38.0%	43.0%	Strength
Overall Average	50.8%	51.8%		60.0%	61.6%	

*Judgements for 1978 have been collapsed into three categories as was done for reporting in 1974.

Relative performance increased on the following four clusters from 1974 to 1978: *Computation With Fractions* (C2), *Computation With Decimals* (C3), *Measurement Systems* (M1), and *Trigonometry* (T1).

Student performance remained a need from 1974 to 1978 on cluster SP1, *Probability and Statistics*, and on two clusters performance remained at an acceptable level from 1974 to 1978, *Application of Geometry* (G2), and *Interpreting Graphs, Maps, and Tables* (I1).

Overall, relative performance from 1974 to 1978 improved on four clusters, remained at the same level on seven clusters, and declined on three clusters. Cluster SP1, *Probability and Statistics*, was the only area to remain a continuing need from 1974 to 1978. No other systematic patterns appear to emerge from the data.

RECOMMENDATIONS FOR IMPROVING MATHEMATICS INSTRUCTION

The following recommendations are suggested by the data, but to some extent have been supplemented by additional observations by selected math educators representing the Minnesota Council of Teachers of Mathematics.

Since the teacher is the key to effective instruction and motivation in the classroom, the following are intended to improve teacher skills:

a. Teacher training must include exposure to a variety of instructional materials needed to build understanding of mathematical concepts. This preparation should include experience with models, charts, films, games, kits, demonstration devices, measuring instruments, and computers.

b. Teacher training must insure that instruction results in the understanding of mathematical concepts. Such preparation should show teachers how to use a learning sequence to help develop understanding in a planned, sequential program.

c. Teacher training should provide an understanding of the use of experiences and materials which can be used to illustrate applications of mathematics. Whenever possible, instruction and assignments should relate the mathematical topic to real life situations in the school and in the community. Field trips, out-of-class assignments, and experiences in local businesses can add variety and significance to mathematics instruction.

d. Teachers should be given experiences and materials to assist them in teaching problem solving. Problem solving is a difficult skill to teach, but it is one of the most important objectives of mathematics instruction.

e. Instruction and practice should be provided for teachers on the use of a calculator or computer in mathematics instruction.

ANALYSIS OF ERRORS ON MATHEMATICS EXERCISES

From an analysis of errors on the eleventh grade mathematics assessment, the following conclusions seem justified:

1. Long division appears to be a difficult area for eleventh graders.

2. The concept of variation is of sufficient importance to require more time and emphasis.

3. It is important that symbols and variables be used in such a manner that operations and variables are not confused. For example, symbols of operation such as the variable "x" may be confused with the operation symbol for multiplication.

4. One of the most important topics in mathematics is functions and relations. Although simple linear equations are solved at a high level, students have great difficulty with inequalities and equations with absolute values. Students also need to attain a higher level of accuracy in using equations to solve problems.

5. Students have considerable difficulty with polynomials, products of binomials, and factoring. This is to be expected because of the difficulty of these items, the limited number who have studied these topics and the limited applications in daily life. Perhaps these topics should be presented at a later age, namely, tenth year geometry or second year algebra. This is also true for quadratic equations and systems of equations.

6. The geometric terms, drawings, and formulas are relatively well understood by eleventh grade students. However, there is a great gap between the identification and use of terms or formulas and the ability to apply this knowledge to life situations. This is particularly true at the 3-dimensional level. Since the world around us is 3-dimensional, perhaps much more attention should be given to this aspect of geometry.

7. Students have attained a high level of proficiency in making measurements, identifying units of measure and comparing units within a given system.

8. From the responses to graphs, tables, and charts, this is an area of high achievement. Since these skills have high application for most citizens, they should continue to receive emphasis at least at the present level.

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9. Students have great difficulty in solving word problems. From an analysis of the test items, it is evident that much of this difficulty lies in the statement of the problem or in the figures given. Often the problem is similar to the items on intelligence tests and has a limited relationship to a student's experiences. When a problem is written for students, it is important to analyze the statements made for clarity, interest, and relevance to the student's experience level.

Summary

An analysis of mathematics errors by eleventh grade students indicates a general high level of achievement in those areas that have relevance to students' daily lives. Abstract mathematical ideas with limited application or complex symbols and drawings generally had a low percent of correct responses. One of the thrusts of mathematics education should be to provide more relevant application of complex mathematical concepts in order to develop greater student mastery.

APPENDIX

Section A

Grade Four

SECTION A1
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 4

SUBOBJECTIVE/DESCRIPTOR	NO. OF ITEMS	MN PERF.	EVALUATION
IA1 Number Facts-Addition	8	94.0%	Strength
IA2 Number Facts-Subtraction	9	93.9%	Strength
IA3 Number Facts-Multiplication	8	91.6%	Strength
IA4 Number Facts-Division	5	85.3%	Strength
IB1 Numeration-Word	6	92.5%	Strength
IB2 Numeration-Symbols	2	93.6%	Strength
IB5 Recognize Examples	9	83.4%	Acceptable
IC1 Recognize Figures	4	86.0%	Potential Strength
ID1 Identify Metric Units	2	68.5%	Strength
ID2 Identify Yards, Feet, Etc.	2	68.6%	Potential Strength
ID3 Identify Coins	6	91.8%	Potential Strength
IE1 Recognize Symbols	7	72.2%	Potential Strength
IIA1a Add Whole Numbers	5	82.5%	Potential Need
IIA1b Add Regrouping	2	84.4%	Strength
IIA2 Subtract Whole Numbers	5	70.0%	Potential Strength
IIA3 Multiply Whole Numbers	4	72.6%	Potential Strength
IIA4 Divide Whole Numbers	4	31.0%	Need
IIA5 Round to 100	1	56.1%	Acceptable
IIA6 Find Sum or Product	1	68.6%	Potential Strength
IIA8 List Consecutive Numbers	1	74.3%	Potential Strength
IIA9 Count By 2, 3, 5, 10	3	88.7%	Potential Strength
IIC1 Measure With Ruler	2	90.4%	Strength
IIC2 Indicate Time	2	81.2%	Strength
IIC3 Write Time	1	77.3%	Strength
IIC4 Read Temperature	3	56.4%	Acceptable
IIC5 Sum Money Values	2	75.8%	Potential Strength
IIC6 Select Coins	2	74.4%	Potential Strength
IIC8 Make Change	1	75.4%	Strength
IID1 Read Graph	4	90.2%	Strength
IID5 Choose Graph	1	43.4%	Strength
IIIA3 Common Properties-Addition	1	75.1%	Potential Strength
IIIA5 Adding Zero	2	94.6%	Potential Strength
IIIA6 Common Properties-Multiply.	1	95.7%	Strength
IIIA8 Multiply By One	2	89.1%	Potential Strength
IIIA9 Greater/Less	2	76.0%	Strength
IIIA10 Select No Sentences	3	65.8%	Potential Strength
IIIC1 Place Value	2	87.4%	Potential Strength
IIIC2 Tell Time	1	82.2%	Potential Strength
IIIC3 No Sentence-Addition	3	90.8%	Potential Strength
IIIC4 No Sentence-Subtraction	1	85.0%	Strength
IIIC5 Compare Sets	1	83.7%	Potential Strength
IIIC6 Add and Multiply	2	81.0%	Strength
IIIC12 Identify Fraction	4	46.8%	Potential Strength
IIIC13 Represent Fraction	1	79.2%	Strength
IIIC16 Check Add and Subtraction	1	53.0%	Potential Need

(Continued)

SECTION A1 (Con't)
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 4

SUBOBJECTIVE/DESCRIPTOR		NO. OF ITEMS	MN PERF.	EVALUATION
IIIC17	Identify Names	2	81.6%	Potential Strength
IIID1	Select Metric Unit	1	75.5%	Strength
IVA1	Solve Word Problem	13	51.8%	Strength
IVB3	Find Volume	2	55.5%	Potential Strength
IVC1	Make Change	2	67.7%	Strength
VA1	Estimate Answer	1	72.8%	Strength
VA2	Order Numbers	2	89.2%	Potential Strength
VA4	Complete No Pattern	4	57.2%	Potential Strength
VB2	Solve Problem	1	84.6%	Potential Strength
VD2	Identify Difference	1	72.3%	Strength

SECTION A2
CLUSTER DEFINITIONS
GRADE 4

The definition and general content of each of the eleven clusters in the fourth grade mathematics assessment are as follows:

B1: Basic Facts of Arithemetic: Recall basic facts of addition, subtraction, multiplication, and division with whole number addends less than 11 and whole number factors through 5 x 9.

C1: Computation with Whole Numbers: Addition, subtraction, multiplication, and division involving whole numbers up to three digits. Also included is the use of multiplication and addition tables to find sums and products of any two whole number entries.

D1: Recognition of Terms and Symbols: Recognize number words to 100, number symbols to 1,000, operation symbols and words, odd and even numbers, units of measure and order relationships between whole numbers.

F1: Introduction to Fractions: Recognize fractional parts less than 9 and order among fractions.

G1: Recognition of Geometric Properties: Recognize names and properties of square, rectangle, triangle, circle, right angle, intersecting and parallel lines. Select appropriate units to determine perimeter, area, and volume.

G2: Applications of Geometric Properties: Identify differences among figures. Compute perimeters of rectangles, triangles, and other polygons.

M1: Measurement Systems: Recognize appropriate units of measure for time, length (English), temperature (F), and money values less $1.00. Read and record time to the quarter hour and temperature. Select correct set of coins from a picture for change for a purchase less than or equal to $1.00.

MM1: Metric Measurement: Select appropriate metric units for measure of length or distance.

P1: Properties of Numbers: Identify place value through 1,000 and rewrite number in expanded notation. Demonstrate commutative property of multiplication. Skip count by 2, 3, 5, and 10. Recognize several different names for the same number and identify the properties of 0 and 1.

PS1: Basic Problem Solving: Translate word problems into mathematics sentences and solve. Also included are exercises which require making change for a dollar for purchases less than $1.00.

SECTION A2 (Con't)
CLUSTER DEFINITIONS
GRADE 4

PS2: Patterns, Logic, and Advanced Problem Solving: Recognize mathematical patterns (ordering) and interpret data from a graph. Estimate answers within one order of magnitude and select the "best buy".

SECTION A3
STUDENT INFORMATION
GRADE 4

Directions: We would like you to give us information about yourself by answering these questions. However, you do not have to do so, if you don't want to. If you do not want to answer any question, you may leave the spaces for that answer blank.

Performance*

Regardless of the grades you make, how smart do you think you are in comparison with the other students in your classes or grade? (Fill in only one circle.)

a.	Not as smart.	67.0%
b.	About as smart as the others.	71.9%
c.	Smarter than the others.	83.8%

How well can you do mathematics?

a.	Poor	63.2%
b.	Fair	72.0%
c.	Good	78.6%
d.	Excellent	81.9%

How do you feel about math compared to the other subjects you study in school?

a.	Math is not one of my favorites.	73.9%
b.	Math is O.K.	77.7%
c.	Math is my favorite subject.	79.1%

Have you ever studied about the metric system in school?

a.	Yes	80.3%
b.	No	74.6%
c.	I don't know.	72.9%

Look at the following list of things which you might have used in a math class. Count the number of those which you remember using in all your school years. Then fill in the circle for your answer.

Abacus
Attribute blocks
Calculator
Compasses
Computer terminal
Flash cards
Geoboard
Math balance
Protractor
Ruler
Slide rule
Tangrams
Dice
Math games
Other devices

a.	I used none of them.	72.1%
b.	I used 1 to 6 of them.	78.1%
c.	I used 7 to 9 of them.	79.2%
d.	I used 10 to 15 of them.	75.2%

*Overall percent correct is reported for students selecting each answer choice.

SECTION A3 (Con't)
STUDENT INFORMATION
GRADE 4

	Performance*
In your mathematics work in school, how much do you use a calculator?	
a. Not at all.	78.7%
b. Only once in a while.	76.8%
c. For everything except tests.	69.5%
d. For everytning.	59.5%
Sex:	
Boys	77.0%
Girls	77.8%

*Overall percent correct is reported for students selecting each answer choice.

SECTION A4

MINNESOTA EDUCATIONAL ASSESSMENT PROGRAM

Mathematics Objectives

Fourth Grade

1978

Key to marginal notations:

90% Objectives which should have been introduced to all the students by the end of their fourth grade year. It is expected that 90% of the students should have mastered the objective and would answer matched items correctly.

50% Objectives which have been introduced to at least half of the fourth grade students by the end of their fourth grade year. It is expected that 50% of the students should be able to correctly answer matched items.

10% Objectives which probably have been introduced only to the best students of mathematics. It is expected that only 10% of the students should be able to correctly answer matched items.

Fourth Grade Mathematics Objectives - 1978

(to be tested in the middle of the fourth grade year)

I. Recall and/or recognition of definitions, fact and symbols.

A task assessing this ability will require only that the examinee be able to recognize typical mathematical symbolism, definitions, and examples of terms of mathematics, or to recall specific facts. It is the lowest level of cognitive ability in mathematics, but is an essential aspect of achievement. Difficulty level in this category will depend more on exposure to the material and on memory than on developed skill.

A. Facts of arithmetic.

The student will:

90% 1. Recall the basic facts of addition with addends less than 11.

90% 2. Recall the basic facts of subtraction, $c - b = a$, where $a, b < 11$.

90% 3. Recall the basic facts of multiplication, $a \times b = c$, where $a < 6$ and $b < 10$.

50% 4. Recall the basic facts of division, $c \div a = b$, where $a < 6$, $c < 46$.

B. Numeration.

The student will:

90% 1. Identify by written word, the counting numbers to 100.

90% 2. Identify by numerical symbol, the counting numbers to 1000.

10% 3. Read and write Roman numerals to XII.

90% 4. Identify by written word and numerical symbol, the unit fractions with denominators of 2, 3, 4, 5, 6, and 8.

90% 5. Recognize examples of the following: whole numbers, equality, addition, subtraction, multiplication, division, sum, difference, product, place value, odd, even, units of measure, and one, two,. . . nine.

50% 6. Recognize examples of the following terms: divisor, quotient, remainder, addend.

90% 7. When given a drawing of a number of objects fewer than 10, will correctly identify the cardinality of the set of objects.

C. Geometry.

The student will:

90% 1. Recognize examples of: square, rectangle, triangle, circle, right angle, intersecting lines, parallel lines.

90% 2. Identify the following by matching the name with an appropriate pictorial representation of the concept: point, line, curve (open and closed), cube.

90% 3. When given a closed curve and a point, identify whether the point is inside, on, or outside the curve.

D. Measurement.

The student will:

50% 1. Given a meterstick, identify a meter and centimeter.

90% 2. Given a yardstick, identify an inch, a foot, and a yard.

90% 3. Given pictures of coins, identify a penny, nickel, dime, and quarter, and tell its value in cents.

50% 4. Identify the quantitative relationship among: centimeter and meter; inch, foot, and yard; gram and kilogram; ounce and pound; second, minute, hour, day, month, and year; and units of U.S. currency.

10% 5. Identify illustrations of conditions under which the Celsius temperature is 0°C, 20°C, 37°C, and 100°C.

E. Algebra.

50% 1. The student will recognize proper use of the symbols: ( ), =, ≠, $<$, and $>$.

F. Sets.

The student will:

10% 1. Recognize examples of equivalent and non-equivalent sets.

10% 2. When given two sets, identify elements which are in both sets.

10% 3. Identify examples of the empty set.

II. Perform mathematical manipulations.

The second level of ability will be assessed by means of tasks which require the examinee to carry out single operations or procedures (or sequences of these) that have been previously learned and are specifically requested. Such tasks will require developed skill, but will not require any decision as to which process or sequence of processes is needed, e.g., algorithm. It is in this category that all straightforward computation is included. It also includes solution of equations. In any case, the tasks which the examinee is required to perform involve only the rote application of learned techniques.

A. Computation and numeration.

The student will demonstrate the following abilities without the use of a calculator:

1. Addition (both horizontal and vertical formats).

90% a) Two whole numbers, each less than 1000, with and without regrouping.

50% b) Three whole numbers each less than 1000, with and without regrouping.

50% 2. Subtraction (both horizontal and vertical formats).

Find the difference between two numbers less than 1000, with and without regrouping.

50% 3. Multiplication.

Name the product of any two digit number and any one digit number less than 6, with and without regrouping.

10% 4. Division.

When given a dividend less than 50 and a one digit divisor, name the quotient and remainder.

90% 5. When given any number less than 1000, correctly round it off to the nearest hundred or the nearest ten, as requested.

90% 6. When given a table of addition or multiplication of two sets of whole numbers, find the sum or product of any two numbers in the sets.

50% 7. When given a rectangular array of numbers, identify the element in the the n-th row and the k-th column.

90% 8. List ten consecutive even or odd numbers starting with any counting number less than 1000.

90% 9. Count by 2's, 3's, 5's, or 10's, starting with 2, 3, 5, or 10, respectively, for at least ten numbers.

B. Algebra.

The student will:

90% 1. Given a blank number line, correctly place the numbers 0-10 on it.

10% 2. Given a simple equation (one variable, one operation), graph its solution on a number line.

50% 3. Given a simple equation, select its solution from a specified domain, i.e., $\{1, 2, 3, 4, 5\}$; which one solves $n + 6 = 10$?

C. Measurement.

The student will:

90% 1. When given a ruler marked off in arbitrary units, measure a given segment to the nearest unit.

90% 2. When given a picture of a clock face without hands, correctly draw the hands to indicate a given time to the five minutes.

50% 3. When given a clock face with a time indicated, will write the time in the form 6:45 to the nearest quarter-hour.

50% 4. Correctly read the temperature indicated on a thermometer.

90% 5. When given pictures of groups of pennies, nickels, dimes, and quarters (some of each), totaling less than \$1.00, name the total value in cents.

90% 6. When given a written amount of money less than $1.00, select coins totaling that amount in value.

90% 7. When given common daily occurances, label the times of those occurances as a.m. or p.m.

90% 8. For a purchase less than $1.00, select the set of coins, from four given sets, which would make the correct change.

D. Probability and statistics.

The student will:

50% 1. When asked questions about a given bar graph or pictograph, name the correct data to answer the question.

10% 2. Tell what one might most likely draw when choosing from a set of different numbers (without looking) of 2 or more kinds of objects, e.g., 7 red and 3 green balls.

10% 3. Record the outcomes of probability events in the form "3 out of 5."

10% 4. After observing a probability experiment and its outcome, give the most likely outcome in a repetition of the same experiment.

10% 5. When given a set of data and possible graphs to illustrate the data, will choose the graph which represents the data.

III. Understand mathematical concepts and processes.

This third level will include tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another. Since the demonstration of comprehension must involve communication, this level will assess comprehension through tasks which include the following possible kinds of translations or transformations within a mathematical context:

--verbal to mathematical, e.g., words to symbols.

--mathematical to verbal, e.g., symbolism to verbal.

--mathematical to mathematical, e.g., translating from one kind of representation to another like an equation to a graph of the equation.

--mathematical to physical, e.g., use of charts to explain fractions.

--physical to mathematical, e.g., developing formulae for physical phenomena.

--verbal to verbal, e.g., explanation.

A. Algebraic expressions.

The student will:

90% 1. Explain the meaning of number frames such as □ and △.

50% 2. When given a verbal statement like "If a number is increased by 5, the result is 12" will write a mathematical sentence such as $N + 5 = 12$.

90% 3. Demonstrate the commutative property of addition by correctly indicating that the commuted form of a statement is equivalent to the original, i.e., $5 + 7 = 7 + 5$.

50% 4. Recognize applications of the associative property of addition by indicating that two sums, arrived at by regrouping the addends, are equal.

90% 5. Give the sum of any number and zero as the given number.

90% 6. Demonstrate the commutative property of multiplication by correctly indicating that the commuted form of the factors give the same result as the original form.

50% 7. Recognize applications of the associative property of multiplication by indicating the equality of products of three one-digit factors, associated in two ways.

90% 8. Give the product of any number and 1 as the given number.

90% 9. Given any two counting numbers less than 1000 in a certain order, write $<$, $=$, or $>$, whichever is the correct symbol, between the two.

50% 10. When given a simple word problem, select from a number of alternatives, the number sentence that correctly fits the problem.

B. Graphing.

The student will:

50% 1. Correctly select from three or more choices, the coordinates of a point with integral coordinates in the first quadrant.

50% 2. When given the integral coordinates of a point in the first quadrant, correctly plot the point.

C. Numeration.

The student will:

90% 1. When given any numeral less than 1000, assign the correct place value to each of the digits.

10% 2. When given any numeral less than 1000, rewrite the number in expanded notation (not exponential form).

90% 3. Correctly match number sentences with set combination drawings (addition).

90% 4. Correctly match number sentences with set and subset removal drawings (subtraction).

90% 5. Identify "how many fewer objects" in one set than in another.

50% 6. Indicate that repeated addition can be done by multiplication.

50% 7. Indicate, without counting, the number of symbols in a rectangular array (less than 25 objects).

90% 8. Find the area of rectangles with integral sides less than 6 units with the unit squares drawn in.

10% 9. Correlate a division statement with a drawing showing the partitions of a set.

10% 10. Divide a given length into a given number of equal parts.

50% 11. When given a set of 30 or fewer objects, name the number of equivalent sets of cardinality 5 or less and the number "left over."

50% 12. When given a drawing of a whole divided into 8 or fewer equal parts, identify the fractional part which is shaded.

50% 13. When given a fraction with a denominator less than 9, correctly shade the corresponding parts of a given region in a drawing.

50% 14. Correctly match number sentences with addition arrows on a number line.

10% 15. Correctly match number sentences with subtraction arrows on a number line.

10% 16. Demonstrate that he/she can check addition by subtracting and vice versa.

50% 17. Correctly identify different names for the same number, e.g., 8, 3 + 5, 12 - 4, etc.

90% 18. When given a sequence of fewer than 10 symbols, name the ordinal position of any of the symbols.

D. Measurement.

The student will:

10% 1. When given examples of different objects whose length or distance is to be measured, select the appropriate metric unit to use.

10% 2. When given various items to find the mass of (to be weighed), select the appropriate metric unit to use.

IV. Solving mathematical problems--social, technical, and academic.

Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem. The common characteristic of tasks in this category will be that they require the individual to analyze a problem and determine a sequence of steps which will lead to a clearly specified outcome (whether the outcome is finding the cost of a purchase or proving a theorem).

A. Reasoning in arithmetical problems.

The student will:

90% 1. When given a simple word problem, find the solution.

10% 2. When given a problem situation and a set of simpler problems, similarly stated, one or more of which is solved in the same way as the original problem, select those alternatives which will help solve the original problem.

50% 3. Determine the distance traveled from two odometer readings and the change in temperature from two above-zero thermometer readings.

50% 4. When given a problem with extraneous data, correctly identify that data as extraneous.

50% 5. When given a problem with insufficient data, correctly identify data which is needed to solve the problem.

B. Generalizations in geometry.

50% 1. The student will find the perimeters of rectangles, triangles, and other polygons.

50% 2. The student will choose the appropriate unit needed to determine perimeter, area and volume.

C. Money problems.

50% 1. The student will make change from a dollar for a purchase less than a dollar.

V. Using mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions, and verify results.

This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Tasks in this category include those which assess the ability to transfer and utilize knowledge in new situations, to recognize patterns, to draw conclusions from given data, to plan for the future on the basis of present information, and to use mathematical reasoning to make optimum decisions.

Tasks in this category also include the ability to recognize the existence of a problem, the ability to state it formally, the ability to formulate hypotheses, and the ability to ascertain if the problem has a unique solution.

A. Patterns and generalizations.

The student will:

10% 1. When given a computation or a problem, and asked to estimate the answer, give an estimation within one order of magnitude.

90% 2. When given a set of five different counting numbers less than 1000, arrange them from smallest to largest.

50% 3. When given any set of unit fractions with denominators less than 9, arrange them from smallest to largest.

50% 4. When given the first few (as necessary) elements from the whole numbers in a number pattern, correctly give the next one, two, or three elements as requested.

10% 5. When given enough ordered pairs of a relation to determine the rule, select another pair (from a group of pairs) that belongs to the relation.

50% 6. When given sets of attribute blocks or pictures of sets with differing attributes, identify the common attribute(s).

B. Consumer buying.

50% 1. When given two identical items at different prices in different stores, the student will select the lower priced item as the best buy.

C. Problem solving.

The student will:

10% 1. When given a problem, for which a prerequisite for solution is the solution of simpler problems, correctly identify the simpler problems necessary for solving the original problem.

10% 2. When given an appropriate problem, present more than one possible solution, e.g., how could you estimate the number of pieces of macaroni in a bag without counting every piece?

D. Geometric relations.

The student will:

50% 1. When given pairs or sets of geometric figures, identify similarities among the figures, such as number of sides, curved boundaries, etc.

50% 2. When given pairs or sets of geometric figures, identify differences among the figures, such as size, shape, number of sides, etc.

10% 3. When given an initial position of a geometric figure, choose the resulting figure, from a set of figures, after a rotation, a reflection, or a translation has been indicated.

10% 4. When given a set of figures, identify those which exhibit line symmetry.

VI. Appreciation and use of mathematics.

The student will:

A. Recognize the importance and relevance of mathematics to the individual and to society.

From age 9 and up, there should be a recognition of the importance and relevance of mathematics to the individual and to society. This subobjective does not necessarily involve enjoyment of mathematics or participation in the development of ideas, but rather it focuses on the acceptance of mathematics as being worthwhile--i.e., the individual recognizes that mathematics is necessary whether or not he/she uses it or enjoys studying it. For example, the individual should recognize the contribution that mathematics has made to the progress of civilization, especially the sciences. There should also be the appreciation of the elegance, economy, and techniques of mathematics. Of course, the level of sophistication of such appreciations should increase with age; nevertheless, some manifestation of these attitudes should appear at all age levels.

B. Recognize that mathematics can be enjoyable.

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques, e.g., using compasses and working with numbers. Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important--e.g., the individual should not hate nor fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study and, therefore, have at his/her disposal.

C. Use the content and techniques of the mathematics he/she has learned.

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participate in the learning of mathematics beyond that which is merely required and actively seek further personal development in the area of mathematics.

The fourth subgoal related to the individual's development of a curiosity about mathematics as well as a readiness to engage in activities in this area, i.e., independent of school and/or job assignments. In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his/her skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgment or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry into adult life.

APPENDIX
Section B
Grade Eight

SECTION B1
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 8

SUBOBJECTIVE/DESCRIPTOR		NO. OF ITEMS	MN. PERF.	EVALUATION
IA	Number Facts	12	95.8%	Strength
IB1	Read and Write Numbers	3	76.2%	Potential Need
IB2	Recognize Terms	2	78.9%	Potential Strength
IB4	Meaning of Symbols	4	82.1%	Potential Strength
IB5	Place Value	1	75.7%	Need
IC1	Geometric Terms	3	73.3%	Acceptable
IC2	Value of PI	1	22.6%	Acceptable
ID1	Metric Units	3	79.0%	Acceptable
ID2	Metric Names	3	59.2%	Potential Need
ID3	Units of Money	4	96.4%	Strength
ID4	Area and Volume	3	52.0%	Potential Need
IE1	Recognize/Use Symbols	6	73.9%	Acceptable
IIA1a	Add Whole Numbers	4	84.3%	Potential Need
IIA1b	Add Decimals	3	66.8%	Acceptable
IIA1c	Add Fractions	2	60.8%	Acceptable
IIA1d	Add Integers	2	40.1%	Potential Need
IIA2a	Multiply Whole Numbers	3	83.8%	Acceptable
IIA2b	Multiply Decimals	3	73.0%	Potential Strength
IIA2c	Multiply Fractions	2	58.7%	Acceptable
IIA2e	Multiply Integers	2	55.2%	Potential Need
IIA3a	Divide Whole Numbers	2	67.5%	Acceptable
IIA3b	Divide Decimals	1	48.5%	Acceptable
IIA4	Decimal Fractions	3	69.7%	Acceptable
IIA5	Decimal Percent	3	50.1%	Potential Need
IIA6	Fraction Equivalent	4	48.4%	Need
IIA7	Order Decimals	3	76.0%	Acceptable
IIA8	Compare	1	53.7%	Potential Need
IIA9	Rounding Numbers	1	64.4%	Potential Need
IIA13	Find Percent	3	23.5%	Need
IIA14	Rename Fractions	4	71.7%	Potential Strength
IIA18	Order Fractions	1	5.5%	Need
IIB1	Use Equation	2	64.0%	Acceptable
IIB3	Simple Equations	1	44.0%	Acceptable
IIB7	Proportion	1	20.6%	Potential Need
IIC1	Measuring	4	81.7%	Acceptable
IIC2	Area/Volume/Perimeter	4	60.3%	Potential Need
IIC4	Convert Measurement	1	23.0%	Need
IID4	Find Average	2	55.5%	Potential Need
IID5	Interpret Graphs	7	80.4%	Potential Strength
IIIA1	Algebra Expression	2	64.3%	Acceptable
IIIB1	Plot Points	2	28.5%	Potential Need
IIIB2	Ordered Pairs	6	64.9%	Acceptable
IIID2	Flow Charts	1	75.8%	Acceptable
IVA1	Verbal Problems	6	59.5%	Acceptable
IVA2	Irrelevant Data	2	54.7%	Acceptable

(continued)

SECTION B1 (Con't)
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 8

SUBOBJECTIVE/DESCRIPTOR		NO. OF ITEMS	MN PERF.	EVALUATION
IVA3	Insufficient Data	3	59.3%	Potential Strength
IVA6	Estimate Results	3	41.9%	Potential Need
IVA7	Estimate Large Numbers	2	53.3%	Acceptable
IVC4	Area and Perimeter	3	37.8%	Potential Need
IVC5	Compute Volume	1	59.6%	Acceptable
IVC7	Size of Angles	1	52.0%	Acceptable
IVD1	Money Totals	2	54.2%	Potential Need
IVD2	Making Change	2	80.7%	Acceptable
IVE1	Reading Tables	7	62.1%	Acceptable
IVE5	Sales Tax	1	51.9%	Potential Need
IVF1	Map Points	2	93.8%	Strength
VA1	Number Patterns	2	51.2%	Acceptable
VA3	3 Dimensional Drawing	1	78.0%	Potential Strength
VB1	Cost Per Unit	2	43.8%	Acceptable
VB2	Discounts	2	23.6%	Acceptable
VB3	Shopping	2	34.4%	Potential Need
VC2	Travel Schedules	4	78.4%	Potential Strength
VC3	Map Distances	1	52.1%	Potential Need
VD	Solving Problems	2	72.5%	Acceptable
VG	Gathering Data	1	64.8%	Potential Strength
TOTALS		177	65.4%	

SECTION B2
CLUSTER DEFINITIONS

The definition and general content of each of the seventeen clusters in the eighth grade mathematics assessment are as follows:

B1: Basic Facts of Arithmetic and Vocabulary: Recall of basic facts of addition, subtraction, multiplication, and division, where the addends or the factors are whole numbers (0-9). Also read and write numbers in the decimal system.

C1: Computation with Whole Numbers: Recall of basic facts and computation with up to four place numbers using addition, subtraction, multiplication, and division.

C2: Concepts and Computation with Common Fractions: The operations of addition, subtraction, multiplication and division are applied to common fractions. Also included is the recognition of fractional comparisons including ratios.

C3: Concepts and Computation with Decimal Fractions: The operations of addition, subtraction, multiplication, and division are applied to decimal fractions. Also included is the recognition of equivalent forms of decimals using percent and common factors.

C4: Computation with Integers: Addition, subtraction, and multiplication of integers (signed numbers).

D1: Recognition of Terms and Symbols: Integer, rational, prime, odd and even numbers, ratio, sets, square root, order relations.

E1: Estimation and Approximation: Recognize a best estimate or estimate the result within one order of magnitude.

A1: Algebraic Expressions: Substitution and translation of verbal statements into algebraic symbolism.

A2: Algebraic Applications: Algebraic symbolism is applied to several situations in determining solutions to equations.

G1: Recognition of Geometric Properties: Names for shapes; perimeter, area, altitude, diameter, parallel, and other commonly used geometric terms.

G2: Applications of Geometric Properties: Using a basic knowledge of area, perimeter, volume, and other properties of geometric figures, this knowledge is applied to a given situation. Computation is often required.

(Continued)

I1: Interpreting Graphs, Maps, and Tables: Picture graphs, and other presentations of data are to be interpreted and conclusions are drawn.

M1: Measurement Systems: Knowledge of both English and metric systems is applied to common situations. The use of the monetary system is included. Some problems require computation.

MM1: Metric Measurement: Recognition of definitions and applications of the metric system of measurement.

P1: Properties of Numbers: Place value and ordering of numbers.

PS1: Basic Problem Solving: Verbally stated problems requiring a direct translation to a solution involving only arithmetic computation.

PS2: Patterns, Logic, and Advanced Problem Solving: The recognition of a number pattern, drawing conclusions, and more advanced problem situations.

SECTION B3
STUDENT PERFORMANCE BY SCHOOL DISTRICT CATEGORY

Objective		No. of Items	MN State Performance	1 Cities of 1st Class	2 7-county Metro Area	3 2,000+ Students	4 1,000-1,999	5 1-999 Students
1A	Number Facts	12	95.8%	92.6%	96.8%	96.1%	95.0%	95.7%
1B	Numeration	10	78.9%	66.4%	81.3%	80.2%	78.1%	78.9%
1C	Geometry	4	60.7%	49.2%	64.1%	63.9%	57.9%	60.7%
1D	Measurement	13	73.6%	65.6%	76.9%	74.7%	72.1%	73.6%
1E	Symbols	6	73.9%	62.0%	75.5%	79.4%	73.6%	73.9%
IIA	Computation	47	61.3%	48.7%	65.7%	63.0%	58.1%	61.3%
IIB	Algebra	4	48.1%	36.1%	53.1%	50.7%	45.6%	48.1%
IIC	Measurement	9	65.6%	55.9%	69.0%	68.7%	64.4%	65.6%
IID	Probability	9	74.9%	65.7%	78.6%	75.2%	72.5%	74.9%
IIIA	Algebraic Exp.	2	64.3%	51.8%	67.0%	70.7%	62.7%	64.3%
IIIB	Graphing	8	55.8%	49.4%	61.6%	57.2%	49.8%	55.8%
IIID	Computers	1	75.8%	59.6%	81.3%	77.0%	71.3%	75.8%
IVA	Reasoning	16	54.8%	44.4%	58.0%	56.7%	52.4%	54.8%
IVC	Generalizations	5	45.0%	35.0%	48.8%	45.8%	43.7%	45.0%
IVD	Money Problems	4	67.5%	61.2%	69.7%	70.1%	65.5%	67.5%
IVE	Tables	8	60.8%	48.3%	64.8%	62.6%	58.7%	60.8%
IVF	Scale Drawings	2	93.8%	85.3%	95.8%	95.3%	91.5%	94.3%
VA	Number Patterns	3	60.1%	51.5%	63.2%	61.1%	55.9%	60.6%
VB	Consumer Buying	6	33.9%	29.7%	37.0%	34.4%	30.5%	31.9%
VC	Maps	5	73.2%	67.1%	74.6%	74.9%	73.8%	70.8%
VD	Problem Solving	2	75.0%	68.3%	76.3%	76.1%	76.7%	73.7%
VG	Gathering Data	1	64.8%	56.6%	64.6%	67.7%	65.3%	65.8%
Overall Average Performance		177	65.4%	55.5%	68.8%	67.1%	63.2%	65.3%

SECTION B4
STUDENT INFORMATION

Directions: We would like you to give us information about yourself by answering these questions. However, you do not have to do so, if you don't want to. If you do not want to answer any question, you may leave the spaces for that answer blank.

Regardless of your grades, how bright do you think you are in comparison to the other students in your classes or grade? (Fill in only one circle)

		Performance*
a.	At the bottom.	46.4%
b.	Well below average.	52.1%
c.	Slightly below average.	53.0%
d.	Average.	61.2%
e.	Slightly above average.	71.8%
f.	Well above average.	77.2%

What is the highest level of education which you would like to attain in the future? (Fill in only one circle)

		Performance*
a.	Some high school, but leave before graduation.	48.2%
b.	Graduate from high school but not go beyond that.	56.0%
c.	Graduate from high school and then go to a vocational, technical, or business school.	63.3%
d.	Go to a junior college.	62.8%
e.	Go to a four-year college or university.	69.6%
f.	Go to graduate or professional school after college.	73.5%

How would you rate yourself in mathematics ability? (Fill in only one circle)

		Performance*
a.	No good in math.	48.6%
b.	Well below average.	51.1%
c.	Slightly below average.	56.1%
d.	Average.	62.3%
e.	Slightly above average.	71.3%
f.	Well above average.	76.4%

Compared to the other subjects you have studied in school, which of the following statements best describes your feelings about math? (Fill in only one circle)

		Performance*
a.	I don't like math at all.	54.5%
b.	Math is my least favorite subject.	57.3%
c.	Math is not included among my favorite subjects.	65.4%
d.	Math is included among my favorite subjects.	68.8%
e.	Math is my most favorite subject.	68.5%

*Overall percent correct is reported for students selecting each answer choice.

Section B4 (Con't)

In your classes in the seventh and eighth grades, how much are you allowed to use a calculator? (Fill in only one circle)

	Performance*
a. Not at all.	64.7%
b. Only once in a while.	66.2%
c. For everything except tests.	65.6%
d. For everything.	61.1%

Have you ever studied about the metric system in your math classes?

	Performance*
a. No	59.3%
b. Yes	67.0%

Look at the following list of things which you might have used in a math class. Count the number of those which you remember using in all your school years. Then fill in the circle for your answer.

Abacus
Attribute blocks
Calculator
Compasses
Computer terminal
Flash cards
Geoboard
Math balance
Protractor
Ruler
Slide rule
Tangrams
Dice
Math games
Other devices

	Performance*
a. I used none of them.	53.9%
b. I used 1 to 6 of them.	60.9%
c. I used 7 to 9 of them.	67.2%
d. I used 10 to 15 of them.	71.7%

Which of the following best describes your math class this year?

	Performance*
a. We use and follow one textbook.	66.7%
b. We use more than one textbook.	64.7%
c. We mostly work on individual worksheets.	61.2%

SECTION B5
ERROR ANALYSIS

ANALYSIS OF ERRORS ON MATHEMATICS EXERCISES

An analysis of eighth grade student responses to mathematics assessment items is contained in this section. Particular attention is given to the types of errors that occurred most frequently. Test items used as examples are reproduced exactly as seen by participants. The percent of the participants choosing each possible answer choice is also displayed. Reasons for incorrect or error responses are speculative since participants choosing an incorrect response were not interviewed to find out why a particular response was chosen. Nevertheless some of the test items did show a consistent pattern of incorrect responses by a sizable portion of the participants and thus lend themselves to discussion.

It is hoped that this section will allow math educators to examine the kinds of errors students make on various problems and subsequently develop materials and teaching strategies to increase students' skills in specific areas.

Clusters involving the following content areas were included in the analysis of errors on mathematics exercises: computation with whole numbers; concepts and computation with common fractions; concepts and computation with common decimals; properties of numbers; algebraic expressions; application of geometric properties; interpreting graphs, maps, and tables, metric measurement; and problem solving.

INFORMATION

In order to analyze errors, selected test items with the percent of participants choosing each responses are reproduced. Following each test

Participants were generally successful in their ability to solve mixed fraction test items. Most of the error responses that would be expected were not in evidence.

Package/Item 4-6: Find the sum

$$\frac{1}{2} + \frac{1}{3}$$

*$\frac{5}{6}$	52.9%
$\frac{1}{6}$	3.1%
$\frac{2}{5}$	15.7%
$\frac{1}{3}$ or $\frac{2}{6}$	4.8%
Other	20.4%
no answer	2.4%

Failing to find the lowest common denominator was the most frequent error on this type of problem. A response of 2/5 was given by 15.7% of the eighth grade participants while the correct response was given by 52.9% of the eighth graders.

Package/Item 4-8:

What does $\frac{2}{3}$ of 9 equal?

*6	56.0%
$9\frac{2}{3}$	1.7%
3 or $\frac{1}{3}$	7.4%
$\frac{2}{3}$	1.5%
other	15.3%
no answer	17.7%

item displayed is a brief discussion of possible reasons for the error responses or interesting aspects of the response. Correct answers are marked with an asterisk.

Package/Item 3-8: Find the quotient

$$6\overline{)608}$$

*101 1/3 or equivalent	80.1%
101	2.2%
11, 11 1/3 or 11 r 2	2.4%
other	10.7%
no answer	4.1%

The error response most typically seen with this division problem would be 11 or dropping the 0 in the tens place. Only 2.4% of the participants completed the division with an error of this nature. Single digit divisor test items were solved correctly by a high percent of the participants.

Package/Item 4-9: Find the quotient

$$28\overline{)3052}$$

*109	54.5%
19	7.5%
190	0.9%
other	23.5%
no answer	13.1%

The correct solution to this division test item was found by only 54.5% of the participants. A typical error would be dropping the 0 in the tens place and 7.5% of the participants did solve the problem in this manner. Also the no answer response has a high percent (13.1%) of the participants.

Package/Item 3-6: Find the sum

$$2\frac{2}{5} + 5$$

*$7\frac{2}{5}$ or equivalent	69.0%	other	20.6%
$2\frac{3}{5}$	0.7%	no answer	6.4%
$3\frac{2}{5}$	2.9%		

The word "of" may have been the reason for a low percent of the participants finding the correct solution. Interestingly many participants (17.7%) decided not to attempt an answer for this test item.

Package/Item 1-6:

Subtract 4.78 from 17.5

*12.72	51.7%
303 decimal anywhere	10.7%
13.72, 12.82 or 13.82	2.0%
13.28	1.9%
other	25.2%
no answer	8.2%

The most frequent error involved subtracting the second number from the first number without any attention given to the decimal place. A response of 303, with the decimal anywhere, was given by 10.7% of the participants. This test item did include the need for regrouping and over half (51.7%) of the participants were able to correctly use the subtraction algorithm with regrouping.

Package/Item 3-19:

What digit is in the thousand's place in the number in this box?

23 486

a.	2	1.1%
*b.	3	77.2%
c.	4	8.7%
d.	8	2.7%
e.	6	9.5%
f.	I don't know	0.4%

Participants may have seen large numbers written only with commas. Contemporary notation has a space rather than a comma to indicate place value. Error responses seem to indicate students either treating the space as a decimal point (9.5%) or treating the third place value (4) from the right as representing the thousandths place.

Package/Item 3-46:

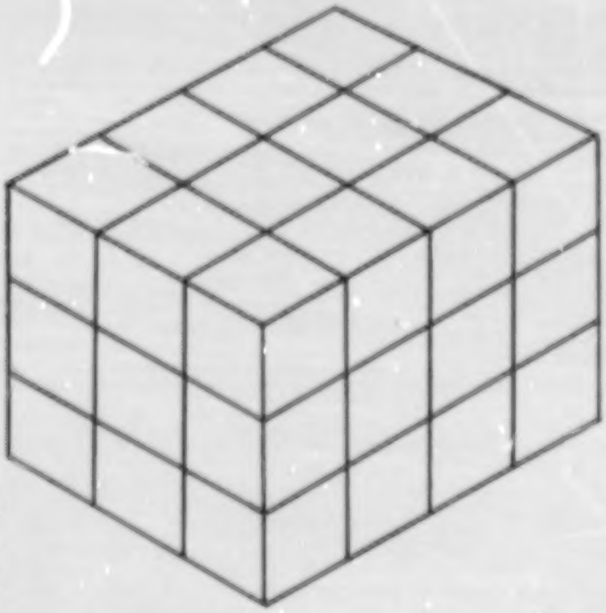
$4 \neq 3$

*a.	True	73.0%
b.	False	17.3%
c.	I don't know	8.6%

Logical thinking with a 'not equal' symbol and a true response may have been the reason some of the participants (17.3%) chose an incorrect response. Prior to this test item there were two true-false items which involved $>$ $<$ symbols and the correct response was chosen by a much higher percent of the participants.

Package/Item 3-13:

A rectangular solid is cut into cubes as shown. How many cubes are there?



*36	37.1%
33	16.5%
66	8.5%
72	2.8%
other	30.0%
no answer	4.7%

Participants (16.5%) giving 33 as a response counted the exposed squares on the diagram rather than the cubes in the solid. On this test item, 41.9% of the boys and 32.1% of the girls identified the correct response.

Package/Item 3-31 and 3-32:

AIRLINE PASSENGERS FOR FIRST SIX MONTHS OF THE YEAR

AIRPORTS	HUNDREDS OF PASSENGERS PER MONTH						TOTAL
	JAN	FEB	MAR	APR	MAY	JUN	
Bay City	9	3	5	7	2	4	30
Camden	6	8	1	5	8	2	30
Dover	8	5	9	6	6	3	37
Fiske	5	6	6	1	3	7	28
Grange	1	2	3	6	7	10	29
TOTAL	29	24	24	25	26	26	154

How many passengers used the Fiske airport in June?

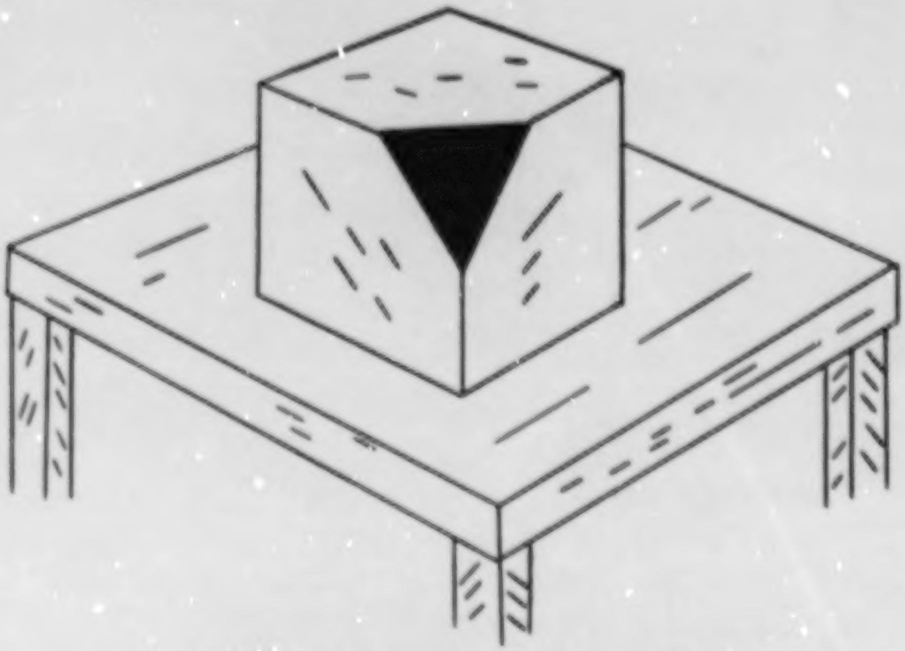
a.	7	61.1%
b.	28	12.8%
c.	70	0.7%
*d.	700	18.3%
e.	2800	4.4%
f.	I don't know	2.0%

How many more passengers used the airports in January than in April?

a.	4	68.3%
b.	5	4.0%
*c.	400	19.0%
d.	500	1.7%
e.	2900	2.7%
f.	I don't know	3.3%

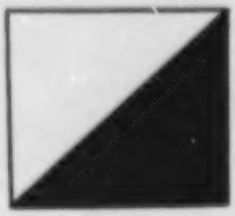
Very few (18.3% and 19.0%) participants read the title in the chart as hundreds. Most likely many of the participants (61.1% and 68.3%) found the numbers in the chart but did not understand what values these numbers represented.

Package/Item 4-34:



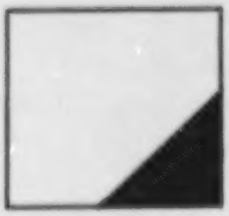
The figure above shows a block with one corner cut off and shaded. Which of the following drawings shows how this block would look when viewed directly from above?

a.



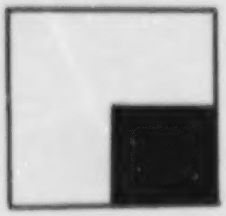
0.8%

*b.



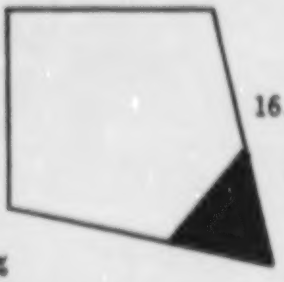
78.0%

c.



3.2%

d.



16.4%

e. I don't know 1.1%

Perceptual difficulty was evident for this test item. Many participants (16.4%) chose figure 'd' as the correct response. On this test item, 85.6% of the boys and 69.7% of the girls gave the correct solution of figures 'b.' Approximately 16 percentage points separated the correct response patterns of the boy and girl participants.

Package/Item 1-20:

In the metric system, the liter is a unit that measures:

*a.	Capacity	64.7%
b.	Area	6.0%
c.	Weight	19.9%
d.	Distance	4.3%
e.	I don't know	4.5%

A liter as a unit of weight was selected by 19.9% of the eighth grade participants while 64.7% of the eighth grade participants chose the correct response of capacity.

Package/Item 3-41, 3-42, and 3-43:

Correct responses related to study of the metric system:

1 liter = 1000 milliliters

a.	True	64.5%	Studied the metric system:	
b.	False	17.7%	Yes	69.8%
c.	I don't know	16.0%	No	46.7%

1 kilogram = 100 grams

a.	True	39.3%	Studied the metric system:	
b.	False	43.7%	Yes	47.9%
c.	I don't know	14.9%	No	30.8%

100 centimeters = 1 meter

a.	True	71.9%	Studied the metric system:	
b.	False	19.1%	Yes	75.0%
c.	I don't know	7.8%	No	61.9%

Relationships within the metric measurement system seemed to be misunderstood by many of the eighth grade participants. As indicated by the data on the right of the page, those eighth grade participants who had studied the metric system more frequently selected the correct response on each test item than those participants who had not studied the metric system.

Package/Item 1-29:

Menu

Hamburger................	.85	Milk.................	.20
Hot Dog..................	.70	Soft Drink...........	.15
Grilled Cheese Sandwich.	.55	Milk Shake...........	.45
French Fries.............	.40	Ice Cream............	.40

Sue had a hot dog, french fries, and milk. How much did she spend?

a.	$1.20	0.8%
*b.	$1.30	95.4%
c.	$1.40	2.1%
d.	$1.50	1.1%
e.	I don't know	0.4%

This test item can be considered a 2-step problem since the price of each menu item must be found and then these prices added together for the total amount spent. Participants from the eighth grade could complete this 2-step operation with familiar price menu items. Most participants (95.4%) selected the correct response. Very few (0.4%) participants claimed they did not know the solution.

Package/Item 4-15:

Sarah paid $1.20 for 6 bottles of cola, including the bottle deposit. If the deposit on each bottle is 5 cents, what is the cost of each bottle of cola?

*a.	15¢	34.0%
b.	20¢	13.3%
c.	25¢	10.8%
d.	30¢	5.6%
e.	other	28.1%
f.	no answer	7.7%

This test item can also be considered a 2-step problem. Only 34.0% of the eighth grade participants found the correct response of 15¢ per bottle. Misunderstanding of the word "deposit" may have been the reason for the many different error responses of 20¢, 25¢, and 30¢.

Package/Item 4-27:

Ms. Baker has between $8000 and $8500 in her savings account. She wants to buy a new car that costs between $5300 and $5400. After she buys the car, how much money will Ms. Baker have in her savings account?

a.	$2700	5.4%
b.	$3100	12.6%
c.	Between $2700 and $3100	61.1%
*d.	Between $2600 and $3200	17.7%
e.	I don't know	2.3%

The selection of 'c' as a response (between $2700 and $3100) showed a misunderstanding by the eighth grade participants of what was meant by a difference. Most of the participants (61.1%) subtracted $5400 from $8500 and $5300 from $8000 to obtain $3100 and $2700 respectively. The concept of difference was translated by the participants into the operation of subtraction rather than understanding the idea of lower and upper bounds of the savings account in relationship to the cost of the new car.

Package/Item 3-35:

A shopper will pay the lowest price per pound for potatoes by buying which of the following:

*a.	10 pounds for 95¢	45.5%
b.	20 pounds for $1.98	23.9%
c.	8 pounds for 89¢	10.3%
d.	5 pounds for 69¢	9.7%
e.	I don't know	9.3%

Many participants (23.9%) willingly believed that the largest quantity has to be the best buy. This was not true for this test item. Also some participants (9.3%) claimed they did not know the solution. Most likely many of these participants were unwilling to solve each possible alternative response to find the lowest price per pound.

SECTION B6

MINNESOTA EDUCATIONAL ASSESSMENT PROGRAM

Mathematics Objectives

Eighth Grade

1978

Key to marginal notations:

90% Objectives which should have been introduced to all the students by the end of their eighth grade year. It is expected that 90% of the students should have mastered the objective and would answer matched items correctly.

50% Objectives which have been introduced to at least half of the eighth grade students by the end of their eighth grade year. It is expected that 50% of the students should be able to correctly answer matched items.

10% Objectives which probably have been introduced only to the best students of mathematics. It is expected that only 10% of the students should be able to correctly answer matched items.

* Those objectives marked with an asterisk are ones which typically are first introduced to students during the eighth grade year, and therefore students may be unfamiliar with them if they are tested during the first half of the eighth grade year.

MEAP Mathematics Objectives
Eighth Grade
1978

I. Recall and/or recognition of definitions, facts and symbols

A task assessing this ability will require only that the examinee be able to recognize typical mathematical symbolism, definitions, and examples of terms of mathematics, or to recall specific facts. It is the lowest level of cognitive ability in mathematics, but is an essential aspect of achievement. Difficulty level in this category will depend more on exposure to the material and on memory than on developed skill.

90% A. Facts of arithmetic

The student will recall the basic facts of addition, subtraction, multiplication, and division, where the addends or the factors are whole numbers, 0 through 9.

B. Numeration

The student will do the following:

90% 1. Read and write numbers in base 10 (thousandths through million).

90%* 2. Recognize examples of the following terms: factor, multiple, decimal, fraction, prime number, square root, percent, ration, integer.

50% 3. Given possible definitions of integer, rational number, and proportion, choose the correct definition of each.

90%* 4. Identify the meaning of the symbols: $\%$, $\frac{a}{b}$, $\sqrt{\ }$, π.

90% 5. Identify the value of the digits of a number written in base 10. (place value)

50% 6. Recognize that fractions and decimals are two forms of a rational number.

10% 7. Recognize the symbols for proportions: $\frac{a}{b} = \frac{c}{d}$, a:b::c:d, and a is to b as c is to d.

10% 8. Read and write numbers in scientific notation.

10% 9. Identify the place value of digits of numbers written in bases other than 10.

10% 10. Recognize the symbols for numbers in bases other than 10, such as 3_{five}.

C. Geometry

The student will do the following:

90%* 1. Recognize examples of the following terms: point, line, plane, segment, angle, polygon, circle, right angle, parallel, perpendicular, horizontal lines, vertical lines, perimeter, diameter, radius, circumference, triangle, square, rectangle, quadrilateral, parallelogram, hexagon, octagon, cube, sphere, cone, cylinder, base, altitude, congruence, similarity.

90% 2. Recall that the value of pi is $3.14\cdots$.

50% 3. Given possible definitions of the following terms, choose the correct one for each: segment, polygon, circle, right angle, parallel, perpendicular, horizontal lines, vertical lines, perimeter, diameter, radius, circumference, triangle, square, rectangle, quadrilateral, parallelogram, hexagon, octagon, cube, sphere, cone, cylinder.

50% 4. Identify the meaning of the symbols: $\angle$, $\triangle$, $\perp$, $\llcorner$, $\parallel$.

D. Measurement

The student will identify:

90% 1. Kilometer, meter, centimeter, and millimeter as units of length; kilogram and gram as units of mass (weight); liter and milliliter as units of capacity; and degrees Celsius as the unit of temperature.

50% 2. The values associated with the metric prefixes: kilo, hecto, deka, deci, centi, and milli.

90% 3. The common money units.

90% 4. The term area as meaning the amount of surface and the term volume as meaning the amount of space enclosed.

10% 5. The differences between standard and non-standard units of measure, e.g., pace and meter, or a square on a piece of graph paper and a square centimeter.

E. Algebra

The student will recognize examples of and correctly use:

90%* 1. Symbols: =, exponent, $>$, $<$, $\neq$, $\geq$, $\leq$.

50% 2. Terms: equation, variable (placeholder), inequality.

F. Sets

The student will identify examples of:

10% 1. Terms: set, element or member, union, intersection, subset, empty (null) set.

10% 2. Symbols: $\{a,b\}$, Venn diagram, $\cup$, $\cap$, $\subset$, ϕ, $\{\}$.

G. Probability and Statistics

The student will:

50% 1. Use the following terms in the context of everyday activities: probability, chance of, odds, frequency, average.

10% 2. Select the correct definitions of these terms: probability, outcomes, arrangements, combinations, mean, median, mode, range, frequency, tally, histogram.

50% H. Technological hardware

The student will identify examples of the following terms: data, data processing, computer, calculator, floating decimal point, readout, keypunching, input, output, real-time computing, batch processing.

II. Perform mathematical manipulations

The second level of ability will be assessed by means of tasks which require the examinee to carry out single operations or procedures (or sequences of these) that have been previously learned and are specifically requested. Such tasks will require developed skill, but will not require any decision as to which process or sequence of processes is needed, e.g., algorithm. It is in this category that all straightforward computation is included. It also includes solution of equations. In any case, the tasks which the examinee is required to perform involve only the rote application of learned techniques.

A. Computation

Note on calculators: The use of calculators in a statewide assessment would not be feasible at this time. Since this is a set of objectives for the Minnesota Assessment Program, we will not include objectives for the use of the hand-held calculator. Teachers should be encouraged to teach the use of calculators and recognize their importance in the program of mathematics education.

The student will demonstrate the following abilities without the use of a calculator:

1. Addition and subtraction

90% a) whole numbers

90% b) decimals

50% c) fractions and mixed numbers (with like denominators and with unlike denominators less than 11).

50% d) integers

2. Multiplication

90% a) whole numbers (one of the factors with 1 or 2 digits).

90% b) decimals (one of the factors with 1 or 2 digits).

90% c) fractions (one of the factors a whole number, the other a fraction or mixed number with numerator and denominator less than 11).

50% d) fractions

50% e) integers

3. Division

50% a) whole numbers (1 or 2 digit divisor, quotient to 2 decimal places).

50% b) decimals (1 or 2 digit divisor, quotient to 2 decimal places).

50% c) integers

50% 4. Given a fraction with the denominator a 1 or 2 digit number, change to a decimal with 2 decimal places.

90% 5. Change a decimal to an equivalent percent and vice versa.

50% 6. Change the following to their fraction equivalents: .5, .25, .75, $.\overline{3}$, $.\overline{6}$, .1, .2, .4, .6, .8 .

90% 7. Arrange decimals and whole numbers according to numerical order.

50% 8. Given two fractions with denominators less than 11, pick out the larger.

90% 9. Round off numerals to nearest ten, hundred, or thousand.

50% 10. Round off decimal numerals to nearest tenth or hundredth.

10% 11. Convert repeating decimals to fractions.

10% 12. Determine the G.C.F., L.C.M. and prime factorization of whole numbers.

50% 13. Solve two types of percentage problems; finding a percent of a number, and finding what percent one number is of another.

50% 14. Rename fractions with equivalent fractions.

90% 15. Find square roots in a table.

10% 16. Find square roots by approximation, division, or algorithm.

B. Algebra

The student will:

90%* 1. Use an equation, such as a = bc or a = b + c, to find the value of one variable given the values of two variables.

50% 2. Recognize examples of the commutative, associative and distributive principles.

50% 3. Find rational number solutions for simple linear equations. ("simple" shall mean one variable written only on one side of the equal sign.)

10% 4. Given a set of data, write the equation that expresses the relationship between two variables.

10% 5. Use a given equation to generate a table of ordered pairs.

50% 6. Determine if two ratios are equivalent.

50% 7. Given three values in a proportion, solve for the fourth.

C. Measurement

The student will:

90% 1. Measure lengths (metric and customary), temperatures, and angles.

50% 2. Find perimeters, areas, and volumes without computing by comparing with standard units.

50% 3. Convert measurements on a map or scale drawing to actual distances.

50% 4. Convert commonly used units of measurement within a measurement system, e.g., g to kg, ft. to in. .

D. Probability and statistics

The student will:

50% 1. Determine the probability of equally likely events, e.g., tossing a coin or a die.

10% 2. Determine the probability of an outcome given a set of data.

10% 3. Compute the mean, median, mode, and range for given data.

90% 4. Given a set of whole numbers and/or decimals, find the average.

90% 5. Read and interpret graphs, such as histogram, circle, bar, and line graphs.

50% 6. Construct an appropriate graph given a set of data.

III. Understand mathematical concepts and processes

This third level will include tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another. Since the demonstration of comprehension must involve communication, this level will assess comprehension through tasks which include the following possible kinds of translations or transformations within a mathematical context:

-- verbal to mathematical, e.g., words to symbols

-- mathematical to verbal, e.g., symbolism to verbal

-- mathematical to mathematical, e.g., translating from one kind of representation to another like an equation to a graph of the equation

-- mathematical to physical, e.g., use of charts to explain fractions

-- physical to mathematical, e.g., developing formulae for physical phenomena

-- verbal to verbal, e.g., explanation.

The student will demonstrate a competency in the following areas:

A. Algebraic expressions

50% 1. Translate a simple verbal expression into an algebraic expression.

10% 2. Represent a mathematical sentence by listing ordered pairs.

10% 3. Translate an equation into a verbal statement.

10% 4. Write a problem situation that represents the relationship expressed by an equation.

10% 5. Find the equation that expresses the relationship in a pattern of data.

B. Graphing

90%* 1. Plot given ordered pairs as points on a coordinate plane.

90%* 2. Give the ordered pair for any point on the coordinate plane.

50% 3. Plot points and intervals on a number line.

50% 4. Write the mathematical statement to describe an interval on a whole line.

10% 5. Represent a simple algebraic equation with an appropriate graph on a coordinate plane.

C. Numeration

10% 1. Write the numerals for numbers 1 to 15 in base two.

10% 2. Write the base ten numeral for a given base two numeral.

D. Computers

90% 1. Develop a flow chart of common everyday occurances.

90% 2. Read a flow chart of common everyday occurances.

50% 3. Develop and read flow charts of mathematical processes.

10% 4. Write a BASIC program for an algebraic process.

IV. Solving mathematical problems -- social, technical and academic

Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem. The common characteristic of tasks in this category will be that they require the individual to analyze a problem and determine a sequence of steps which will lead to a clearly specified outcome (whether the outcome is finding the cost of a purchase or proving a theorem).

A. Reasoning in arithmetical problems

The student will:

90% 1. Find the answer to a verbal problem which involves quantitative relationships familiar to the student, e.g., money, measurement.

50% 2. Find irrelevant data included in a problem.

50% 3. Identify problems in which there are insufficient data given and indicate what data are needed to solve the problem.

50% 4. Find examples that verify a statement such as, "The sum of any two odd numbers is an even number."

50% 5. Find examples that contradict a statement such as, "All prime numbers are odd numbers."

50% 6. Given a familiar problem, will recognize a best estimate of the results.

10% 7. Given "large-scale" problems, estimate the result within one order of magnitude, e.g., population, transportation, etc. .

B. Algebraic problems

The student will:

10% 1. Identify the formula or equation, such as $d = rt$, which is involved in the relationship described in a problem.

10% 2. Solve a problem which is based on a relationship which can be expressed by an equation.

10% 3. Discover the relationship or equation which will solve a problem by considering special cases involving small numbers.

C. Generalizations in geometry

The student will:

50% 1. Make sketches of a geometric situation ("Draw a picture!") to provide a pattern from which a generalization can be stated.

10% 2. Collect data to find a generalization, e.g., "Measure and find the sum of the angles of a polygon."

50% 3. Given a geometric pattern of changes, predict the missing pattern, e.g., △, ◇, __, ⬠, ⬡, ...

50% 4. Given measurements, compute the perimeters and areas of triangles, squares, rectangles, and circles.

50% 5. Given measurements, compute the volumes of cubes and rectangular prisms (boxes).

10% 6. Given formulas and measurements, compute the:

a) areas of trapezoids, parallelograms, regular polygons, etc.

b) volumes of cylinders, pyramids, cones, spheres, etc. .

50% 7. Given two intersecting lines and the size of one of the angles formed, determine the sizes of the other angles in the figure.

50% 8. Given a picture of a bicycle, identify geometric figures.

10% 9. Given two sides of a right triangle, determine the third side.

D. Money problems

The student will:

90% 1. Determine different ways to make totals such as 7¢, 13¢, 19¢, etc. .

90% 2. Given an amount to be paid, make change from a larger amount.

E. Tables

The student will:

90% 1. Read and interpret a table of data.

50% 2. Given a set of data, construct a table.

10% 3. Interpolate to find a value that is not included in a table.

10% 4. Plan an experiment which collects data to determine a relationship, e.g., the amount a rubber band will stretch each time a weight is added.

50% 5. Determine sales tax by using tax tables.

F. Scale drawings and maps

The student will:

90% 1. Locate points on a highway map with ordered pairs.

10% 2. Find the solution to a problem situation by making a scale drawing and measuring the required length, e.g., find the unknown height of a pole.

10% 3. Given certain measurements, determine unknown distances by scale drawing on a graph paper.

10% 4. Given certain measurements, directions and locations of objects in a given area, draw a map.

10% 5. Given a scale drawing, determine distances, directions, and locations.

V. Using mathematics and mathematical reasoning to analize problem situations, define problems, formulate hypotheses, make decisions and verify results

This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Tasks in this category include those which assess the ability to transfer and utilize knowledge in new situations, to recognize patterns, to draw conclusions from given data, to plan for the future on the basis of present information, and to use mathematical reasoning to make optimum decisions.

Tasks in this category also include the ability to recognize the existence of a problem, the ability to state it formally, the ability to formulate hypotheses, and the ability to ascertain if the problem has a unique solution.

A. Patterns and generalizations

The student will:

50% 1. Recognize a pattern in numbers, then extend the pattern and/or generalize the sequence.

10% 2. Recognize a pattern in geometric figures, then extend the pattern, e.g., the number of diagonals from one vertex in a polygon.

50% 3. Visualize a three-dimensional drawing and recognize other views of the same item.

B. Consumer buying

The student will:

50% 1. Determine the cost per unit, e.g., cost of one gram (ounce) if 7.5 grams (ounces) costs 45¢.

50% 2. Determine the discount on sale items.

50% 3. Determine if a savings is made by buying "large economy size" rather than smaller size units.

C. Planning a trip

The student will:

50% 1. Determine the cost per kilometer (mile) or automobile travel considering gasoline, oil, repairs, depreciation, insurance, and taxes.

50% 2. Read travel schedules for plane, train and bus.

90% 3. Determine distances on a highway map for driving.

D. Problem solving

50% The student will solve problems requiring analysis, but at a level of comprehension appropriate for eighth grade students, e.g., determine the number of persons in an auditorium with 24 rows and 12 seats in a row if 37 seats are empty.

E. Novel problems and puzzles

10% The student will solve problems such as the following:

1. Given 4 squares, how many polyominoes can be formed?

2. Which of these networks can be traced without retracing a line and without lifting the pencil off the paper?

3. How many coins can be placed on these grids so there is only one coin on each row, each column, and each diagonal?

4. Find the missing numbers in a magic square.

F. Geometric relations

10% The student will solve problems involving geometric relations such as:

1. Given several 30-60-90 degree triangles, determine the relationship between the shortest and longest sides.
2. Given several quadrilaterals and lines joining the midpoints of adjacent sides, determine the properties of the figure formed by the lines.

G. Conclusions from data

50% The student will answer questions such as the following:

1. If you were to survey the students in your school to find out how many ride a bicycle to school, how would you determine which students to questions?
2. What measures would you make to find out the speed at which you ride your bicycle around the block?

VI. Appreciation and use of mathematics

A. Recognizing the importance and relevance of mathematics to the individual and to society.

From age 9 and up, there should be a recognition of the importance and relevance of mathematics to the individual and to society. This subobjective does not necessarily involve enjoyment of mathematics or participation in the development of ideas, but rather it focuses on the acceptance of mathematics as being worthwhile --i.e., the individual recognizes that mathematics is necessary whether or not he/she uses it or enjoys studying it. For example, the individual should recognize the contribution that mathematics has made to the progress of civilization, especially the sciences. There should also be the appreciation of the elegance, economy, and techniques of mathematics. Of course, the level of sophistication of such appreciations should increase with age; nevertheless, some manifestation of these attitudes should appear at all age levels.

B. Enjoyment of mathematics

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques, e.g., using compasses and working with numbers. Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important -- i.e., the individual should not hate nor fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study and, therefore, have at his/her disposal.

C. Using the content and techniques of mathematics

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participation in the learning of mathematics beyond that which is merely required, and actively seeking to further personal development in the area of mathematics

The fourth subgoal related to the individual's development of a curiosity about mathematics as well as a readiness to engage in activities in this area, i.e., independent of school and/or job assignments. In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his/her skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgement or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry into adult life.

APPENDIX
Section C
Grade Eleven

SECTION C1
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 11

SUBOBJECTIVE/DESCRIPTOR	NO. OF ITEMS	MN PERF.	EVALUATION*
IA1 Number Facts-Addition	6	96.3%	Strength
IB1 Percent and Ratio	1	95.5%	Strength
IB3 Place Value	3	70.5%	Need
IC3 Terms-Lines	8	73.4%	Strength
IC4 Terms-Figures	7	63.4%	Potential Need
IC7 Pythagorean Theorem	2	46.6%	Potential Need
IC8 Formulas	4	56.6%	Acceptable
ID1 Identify Metric Units	3	84.0%	Potential Strength
ID2 Identify Yards, Feet, Etc.	7	58.6%	Acceptable
ID4 Scales in Measurement	3	92.8%	Strength
IE2 Meaning of Symbols	4	88.0%	Strength
IE6 Exponents and Logarithms	2	72.0%	Strength
II1 Sine, Cosine, Tangent	3	22.3%	Acceptable
II2 Functions of Angles	1	6.1%	Acceptable
IJ1 Examples of Gross, Net, Etc.	1	74.2%	Strength
IIA1 Whole Numbers-Add	8	78.2%	Potential Strength
IIA2 Whole Numbers-Multiply	4	78.7%	Acceptable
IIA3 Whole Numbers-Divide	3	67.1%	Acceptable
IIA6 Ordering Numbers	4	73.9%	Potential Need
IIA7 Rounded Numbers	2	79.0%	Potential Strength
IIA8 Percentage Problems	3	67.2%	Potential Need
IIA9 Find Equivalency	8	82.7%	Potential Strength
IIA10 Negative Numbers	4	77.3%	Strength
IIB2 Simplify Grouping Symbols	3	48.0%	Potential Need
IIB3 Quadratic Expressions	1	28.0%	Need
IIB7 Linear Equations	1	25.9%	Acceptable
IIB11 Inequalities	5	41.4%	Potential Need
IIB12 Factoring	2	23.5%	Potential Need
IIC1 Geometric Constructions	1	45.6%	Acceptable
IIC2 Use of Protractor	4	88.9%	Strength
IID1 Conversion Relations	4	63.7%	Acceptable
IID4 Perimeters, Areas, Etc.	2	39.4%	Acceptable
IID5 Correct Change	1	90.0%	Strength
IID6 Scale Maps	1	72.2%	Potential Strength
IIE1 Read Statistical Graphs	4	93.9%	Strength
IIE2 Mean, Median, Mode	6	47.6%	Need
IIE3 Probability	1	65.7%	Strength
IIIA2 Mathematical Sentences	3	53.6%	Potential Strength
IIIB1 Plot Ordered Pairs	6	86.0%	Strength
IIIB2 Graph-Linear Functions	1	27.5%	Potential Need
IIIB3 Graph-Quadratic Function	1	41.0%	Strength

*As determined by a group of Minnesota educators.

(Continued)

SECTION C1 (Con't)
STUDENT PERFORMANCE ON MATHEMATICS SUBOBJECTIVES
GRADE 11

SUBOBJECTIVE/DESCRIPTOR	NO. OF ITEMS	MN PERF.	EVALUATION*
IIIB4 Write Linear Function	1	30.2%	Potential Strength
IVA1 Apply Formulas	4	41.2%	Potential Need
IVA5 Simple Arithmetic Problems	1	82.0%	Potential Strength
IVB1 Linear Equation Problems	2	34.1%	Need
IVB2 Simple Inequalities	1	7.0%	Need
IVD1 Determine Sales Tax	2	86.5%	Strength
IVD2 Find Square Root	4	47.7%	Potential Need
IVD4 Interpret Table, Chart, Etc.	7	61.6%	Acceptable
IVE1 Read Road Map	1	68.6%	Acceptable
IVE2 Locate Points on Map	2	96.8%	Strength
IVG1 Common Household Problems	3	58.4%	Potential Need
IVG2 Solve Consumer Problems	5	54.9%	Acceptable
IVG4 Problems Involving Area	4	23.6%	Need
IVH1 Properties of Numbers	2	40.9%	Need
VA1 Recognize Patterns	2	57.1%	Potential Strength
VB1 Read Travel Schedules	4	80.7%	Strength
VB2 Installment Buying	2	49.1%	Potential Strength
VB3 Determine Discount	1	43.9%	Potential Need
VB4 Comparative Buying	2	20.7%	Need
VC2 Verify Solution	1	33.3%	Need
VC4 Counter Examples, Etc.	2	78.5%	Strength
VD1 Solve Novel Problems	1	25.0%	Potential Strength

*As determined by a group of Minnesota educators.

SECTION C2
CLUSTER DEFINITIONS

The definition and general content of each of the eighteen clusters in the eleventh grade mathematics assessment are as follows:

B1: Basic Facts of Arithmetic and Vocabulary: Recall of basic facts of addition, subtraction, multiplication, and division, where the addends or the factors are whole numbers (0-9). Also read and write numbers in the decimal system.

C1: Computation With Whole Numbers: Recall of basic facts and computation with up to four place numbers using addition, subtraction, multiplication, and division.

C2: Concepts and Computation With Common Fractions: The operations of addition, subtraction, multiplication, and division are applied to common fractions. Also included is the recognition of fractional comparisons including ratios.

C3: Concepts and Computation With Decimal Fractions: The operations of addition, subtraction, multiplication, and division are applied to decimal fractions. Also included is the recognition of equivalent forms of decimals using percent and common factors.

C4: Computation With Integers: Addition, subtraction, and multiplication of integers (signed numbers).

A1: Algebraic Expressions: Substitution and translation of verbal statements into algebraic symbolism including solving equations.

A2: Algebraic Applications: Algebraic symbolism applied to several situations in determining solutions to equations.

G1: Recognition of Geometric Properties: Names for shapes; perimeter, area, altitude, diameter, parallel, and other commonly used geometric terms.

G2: Applications of Geometric Properties: Using a basic knowledge of area, perimeter, volume, and other properties of geometric figures in a given situation. Computation is often required.

I1: Interpreting Graphs, Maps, and Tables: Bar and line graphs and other presentation of data are to be interpreted and conclusions are stated.

SECTION C2 (Con't)
CLUSTER DEFINITIONS

M1: Measurement Systems: Knowledge of both English and metric systems is applied to common situations. The use of the monetary system is included. Some problems require computation with measures.

MM1: Metric Measurement: Recognition of definitions, units, and applications of the metric system of measurement.

P1: Properties of Numbers: Place value and ordering of numbers.

PS1: Basic Problem Solving: Verbally stated problems requiring a direct translation into a solution involving only arithmetic computation.

PS2: Patterns, Logic, and Advanced Problem Solving: The recognition of a number pattern, drawing conclusions, and more advanced problem situations.

PS3: Consumer Problem Solving: Problems related to every day financial situations which require basic computational skills.

SP1: Probability and Statistics: Use of mean, median, and mode in summarizing data. Basic notions of the probability of an event.

T1: Trigonometry: Basic knowledge and applications of the sine, cosine, and tangent functions.

SECTION C3
STUDENT INFORMATION

Directions: We would like you to give us information about yourself by answering these questions. However, you do not have to do so, if you don't want to. If you do not want to answer any question, you may leave the spaces for that answer blank.

	Performance*
Regardless of the grades you make, how smart do you think you are in comparison with other students in your classes or grade?	
a. Not as smart	49.9%
b. About as smart	63.2%
c. Smarter	76.8%
What is the highest level of education which you would like to attain in the future? (Fill in only one circle.)	
a. Some high school, but leave before graduation.	45.4%
b. Graduate from high school, but not go beyond that.	52.9%
c. Graduate from high school and then go to a vocational, technical, or business school.	58.8%
d. Go to a junior college.	63.5%
e. Go to a four-year college or university.	72.0%
f. Go to graduate or professional school after college.	77.6%
How do you feel about math compared to the other things you study in school?	
a. Math is my least favorite.	54.6%
b. Math is not one of my favorites.	61.8%
c. Math is one of my favorites.	72.8%
d. Math is my favorite.	77.9%
How hard is math compared to other things you study in school?	
a. Math is easier than most of the other things.	73.1%
b. Math is about the same as the other things.	66.1%
c. Math is harder than most of the other things.	59.7%
In your math classes in grades 9-11, how much have you been allowed to use a calculator?	
a. Not at all	58.8%
b. Only once in a while	65.3%
c. For everything except tests	66.9%
d. For everything	69.3%

*Overall percent correct is reported for students selecting each answer choice.

SECTION C3 (Con't)
STUDENT INFORMATION

	Performance*
Which one of these best describes your father's education?	
a. He did not attend high school at all.	60.6%
b. He attended high school, but did not graduate.	59.4%
c. He graduated from high school.	63.6%
d. He attended college or some other school after high school.	69.4%
Which one of these best describes your mother's education?	
a. She did not attend high school at all.	60.6%
b. She attended high school, but did not graduate.	57.8%
c. She graduated from high school.	63.8%
d. She attended college or some other school after high school.	69.2%
Which of these categories best describes your father's occupation?	
a. Semi-skilled: general work in a factory, farm, hospital, restaurant, service station, etc. (work that did not require a lot of training.	61.9%
b. Skilled: skilled work in a factory, farm, hospital, mechanic's shop, etc. (work that required a lot of training and experience).	63.5%
c. Clerical & Sales: work in an office, store, business, factory, post office, bank, etc.	63.9%
d. Professional & Executive: work as a doctor, lawyer, engineer, accountant, teacher, manager or person in charge, director, etc.	69.8%
e. Some other.	59.8%
Which of these categories best describes your mother's occupation?	
a. Housewife: does not go out to work for money.	65.0%
b. Semi-skilled: general work in a factory, farm, hospital, restaurant, service station, etc.	61.9%
c. Skilled: skilled work in a factory, farm, hospital, mechanic's shop, etc.	63.5%
d. Clerical & Sales: work in an office, store, business, factory, post office, bank, etc.	66.8%
e. Professional & Executive: work as a doctor, lawyer, engineer, accountant, teacher, manager or person in charge, director, etc.	67.8%
f. Some other.	61.9%
Sex:	
Boys	66.2%
Girls	62.9%

SECTION C3 (Con't)
STUDENT INFORMATION
MATH COURSES TAKEN

Directions: Here is a list of math courses. Indicate when you took the course, or that you did not take it.*

COURSES	WHEN TAKEN				
	Gr. 8	Gr. 9	Gr. 10	Taking Now	Did Not Take
General Math (9th grade)	48.0%	25.0%	3.2%	1.3%	14.0%
Algebra 1 (First-year Algebra)	6.7%	61.0%	11.0%	1.6%	14.0%
Geometry	1.0%	5.6%	39.0%	7.7%	38.0%
Algebra 2 (Second-year Algebra)	.2%	2.1%	12.0%	26.0%	49.0%
Trigonometry	.1%	.6%	4.3%	13.0%	69.0%
Probability and Statistics	.9%	1.0%	1.2%	2.3%	78.0%
Computer Math	1.6%	2.1%	2.3%	5.2%	72.0%
Business, Consumer, or Vocational Math	.7%	1.6%	5.8%	12.0%	63.0%

*Rows may not add to 100% due to non-responses.

SECTION C4

Eleventh Grade Mathematics Objectives - 1979

(to be tested near the end of the eleventh grade year)

I. Recall and/or recognition of definitions, facts and symbols.

A task assessing this ability will require only that the examinee be able to recognize typical mathematical symbolism, definitions, and examples of terms of mathematics, or to recall specific facts. It is the lowest level of cognitive ability in mathematics, but is an essential aspect of achievement. Difficulty level in this category will depend more on exposure to the material and on memory than on developed skill.

A. Facts of arithmetic.

The student will:

1. Recall the basic facts of addition, subtraction, multiplication, and division, where the addends or the factors are whole numbers, 0 - 9.

B. Numeration.

The student will:

1. Recognize examples of percent and ratio, and the symbols % and $\frac{a}{b}$.
2. Read and write numbers (from 1×10^{-9} to 1×10^{9}) in scientific notation.
3. Identify the value of the digits of a number written in base ten (place value).

C. Geometry.

The student will:

1. Identify the meaning of the symbols: $\parallel$, $\perp$, $\triangle PQR$
2. Identify the meaning of the symbols: $\cong$, $\sim$
3. Recognize examples of the following terms: parallel, perpendicular, congruent, similar triangles, segment, ray, horizontal, vertical lines, radius, diameter.
4. Recognize examples of the following terms: polygon, triangle, quadrilateral, parallelogram, rectangle, square, rhombus, trapezoid, pentagon, hexagon, octagon, circle, isosceles triangle, equilateral triangle, right angle.

5. Identify examples of: coordinates, ordered pairs, slope, distance, midpoint.

6. Recognize the size relationships among angles formed by chords, secants and tangents drawn to a circle.

7. Recognize the need to use the Pythagorean theorem when working with right triangles.

8. Recognize mensurational formulas for perimeters, areas and volumes of common geometric figures.

D. Measurement.

The student will:

1. Recognize that the units in the metric system are: for length - meter, centimeter, kilometer; for mass (weight) - gram, kilogram; for capacity - liter, milliliter; and, for temperature - degrees Celsius.

2. Recognize the common units of length, weight, capacity, and temperature in the customary system and the metric system.

3. Recognize the use of scientific units: calorie, BTU, wire gauge, etc.

4. Read rules and scales used in measurement.

E. Algebra.

The student will:

1. Recognize examples of the terms: variable, linear and quadratic equations, coordinates, ordered pair, rational and real numbers.

2. Identify the meaning of the symbols: $<, >, =, \leq, \geq, +, -, \times, \div$

3. Identify the meaning of the symbols: $\cap, \cup, |x|$

4. Identify the meaning of the symbols: $f(x)$, $\log x$, a^x

5. Choose the best definitions of: function, inverse, field.

6. Recognize the laws of exponents and logarithms.

7. Match the graphs and equations of lines, circles, and inequalities.

8. Identify the graphs of: parabola, ellipse, and hyperbola.

F. Sets and logic.

The student will:

1. Recognize the meaning of the symbols: $\wedge, \vee, \rightarrow, \leftrightarrow$
2. Recognize examples of: biconditional, converse, inverse, contrapositive, and counterexample.

G. Probability and statistics.

The student will:

1. Recognize examples of: sample space, probability of success, mean, median, mode, permutations, combinations.
2. Identify the meaning of the symbols: $\Sigma, \bar{x}, !$

H. Technological hardware.

The student will:

1. Identify the functions of the parts of a computer: input, memory, processor, output.
2. Identify the differences in the functions and use of computers and calculators.
3. Recognize differences between human and computer errors.

I. Trigonometry.

The student will:

1. Choose the best definitions of: sine, cosine, tangent.
2. Recognize the values of the trig functions of these angles: 0°, 30°, 45°, 60°, 90°.

J. Consumer topics.

The student will:

1. Identify examples of: gross, net, profit, loss, selling price, cost, discount, simple interest, compound interest, depreciation, unit pricing.
2. Identify the examples of: prorate, tax millage, exemption, debt.
3. Identify differences between: checking and savings accounts, stocks and bonds.

II. Perform mathematical manipulations.

The second level of ability will be assessed by means of tasks which require the examinee to carry out single operations or procedures (or sequences of these) that have been previously learned and are specifically requested. Such tasks will require developed skill, but will not require any decision as to which process or sequence of processes is needed, e.g., algorithm. It is in this category that all straightforward computation is included. It also includes solution of equations. In any case, the tasks which the examinee is required to perform involve only the rote application of learned techniques.

A. Computation.

The student will:

1. Add and subtract whole numbers and decimals with fewer than six digits, and fractions and mixed numbers with like denominators and with unlike denominators less than 11.

2. Multiply whole numbers and decimals where one of the factors has one or two digits, and multiply fractions with denominators less than 11.

3. Divide whole numbers and decimals with 1 or 2 digit divisors, and write the quotient to 2 decimal places.

4. Use the hand held calculator to make the arithmetical computations needed to solve stated verbal problems.

5. Find square roots by using a table.

6. Arrange a group of rational numbers according to numerical order.

7. Round off numbers to the nearest 10, 100, or 1000.

8. Solve two types of percentage problems, finding the percent of a number, and finding what percent one number is of another.

9. Find equivalent representations for given fractions, decimals, ratios and percents.

B. Algebra.

The student will:

1. Combine like terms.

2. Simplify and evaluate expressions which contain grouping symbols.

3. Factor appropriate quadratic expressions.

4. Correctly use logarithms for computation.

5. Correctly evaluate determinants.

6. Perform operations with matrices.

7. Solve systems of linear equations.

8. Solve systems of quadratic equations.

9. Perform synthetic division on polynomials.

10. Expand a binomial to any power less than 5.

11. Solve linear equations and inequalities in one variable, including proportions.

12. Solve quadratic equations by factoring and by formula.

13. Evaluate the discriminant of a quadratic equation and use its value to determine the nature of the roots.

C. Geometry.

The student will:

1. Make the simple geometric constructions: copy segment and angle, bisect segment and angle, perpendicular lines, parallel lines.

2. Use a protractor to find the size of angles.

D. Measurement and mensuration.

The student will:

1. Correctly use conversion relations within a measurement system.

2. Correctly use carpenter and engineering rules to determine length.

3. Find relative error of measurement.

4. Use formulas to compute the perimeters, areas, and volumes of common geometric figures.

5. Determine the correct change for a purchase when given paper money greater than the amount of the purchase.

E. Probability and statistics.

The student will:

1. Correctly read various types of statistical graphs (circle, bar, line, etc.).

2. Compute and interpret mean, median and mode of a group of data.

III. Understand mathematical concepts and processes.

This third level will include tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another. Since the demonstration of comprehension must involve communication, this level will assess comprehension through tasks which include the following possible kinds of translations or transformations within a mathematical context:

--verbal to mathematical, e.g., words to symbols.

--mathematical to verbal, e.g., symbolism to verbal.

--mathematical to mathematical, e.g., translating from one kind of representation to another like an equation to a graph of the equation.

--mathematical to physical, e.g., use of charts to explain fractions.

--physical to mathematical, e.g., developing formulae for physical phenomena.

--verbal to verbal, e.g., explanation.

A. Algebraic expressions.

The student will:

1. Identify linear, quadratic, and exponential functions.
2. Translate a verbal statement into a mathematical sentence.
3. Give examples of: function, slope, ratio, proportion, relation, limit, continuity, inverse.
4. Identify uses of: transforming expressions, graphing, factoring, using formulas.

B. Graphing.

The student will:

1. Plot ordered pairs on a coordinate plane.
2. Draw a graph of given linear functions and inequalities.
3. Draw a graph of a given quadratic function.
4. When given a graph of a linear function, write the function.
5. When given the graph of a second degree function, write the corresponding function used.
6. When given a non-linear graph, will identify roots, maxima and minima of the function.

C. Computers.

1. Make a flow chart of a mathematical process or problem.

2. Recognize the meaning of simple BASIC commands: Print, Read-data, Input, For-next.

IV. Solving mathematical problems--social, technical and academic.

Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information and technqiues which are needed to solve a particular problem and to apply such background in actually solving the problem. The common characteristic of tasks in this category will be that they require the individual to analyze a problem and determine a sequence of steps which will lead to a clearly specified outcome (whether the outcome is finding the cost of a purchase or proving a theorem).

A. Reasoning in arithmetical problems.

The student will:

1. Apply formulas such as $d=rt$, $I=Prt$, and mensuration formulas to problem situations.

2. Solve problems involving arithmetic and geometric series.

3. When given a problem situation dealing with quantitative results, estimate the answer within one order of magnitude.

4. Determine the cost per kilometer (mile) of driving a car, considering gas, oil, repairs, insurance, depreciation, etc.

5. Solve verbal problems involving simple arithmetic.

B. Algebraic problems.

The student will:

1. Solve verbal problems that involve simple linear equations.

2. Solve verbal problems that involve simple inequalities.

3. Solve verbal problems involving second degree equations and inequalities with two variables.

4. Perform all operations with complex numbers.

5. Locate a flaw in an algebraic demonstration.

C. Generalizations in geometry.

The student will:

1. Prove relationships in geometric situations using deduction and indirect methods.
2. Locate a flaw in a geometric demonstration.

D. Tables.

The student will:

1. Determine the amount of sales tax by using a table of sales tax.
2. When given a table of square roots, cube roots, primes and factors, or trig functions, will find values as requested.
3. When given an arithmetic or trigonometric table, will interpolate to find values between table entries.
4. Interpret a table, chart, or graph.

E. Scale drawings and maps.

The student will:

1. When given a road map, determine the highway distance and directions between cities.
2. Locate points on a highway map, when given ordered pairs of letters and numbers used for that purpose.
3. Determine the differences in time between cities in different time zones.

F. Trigonometry.

The student will:

1. Solve for the unknown parts of a right triangle by using trig ratios.
2. Verify simple trig identities.
3. Solve simple navigation problems.
4. Solve simple trig equations.

G. Consumer problems.

The student will:

1. Solve common household problems involving mathematics, such as home repair, sewing, marketing, food preparation, and bookkeeping.

2. Solve such consumer problems as in the areas of budgets, taxes, insurance, checking accounts, etc.

3. Prepare a bank deposit slip when given the needed data.

4. Solve problems involving area at home such as: cost of paint, amount of tiling, size of carpeting.

H. Number properties.

The student will:

1. Apply and generalize properties of numbers.

V. Using mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions, and verify results.

This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Tasks in this category include those which assess the ability to transfer and utilize knowledge in new situations, to recognize patterns, to draw conclusions from given data, to plan for the future on the basis of present information, and to use mathematical reasoning to make optimum decisions.

Tasks in this category also include the ability to recognize the existence to a problem, the ability to state it formally, the ability to formulate hypotheses, and the ability to ascertain if the problem has a unique solution.

A. Patterns and generalizations.

The student will:

1. Recognize patterns and make generalizations about numerical, algebraic, and geometric configurations.

B. Consumer buying.

The student will:

1. Read travel schedules for bus, train, and plane.

2. Determine the cost of installment buying.

3. Determine the amount of discount on sale items.

4. Exhibit problem solving behaviors in comparative buying, planning personal finances, budgeting, and cost estimating.

C. Problem solving.

The student will:

1. Use a computer language to solve original problems.

2. After solving a problem, will verify the reasonableness of solution.

3. When given an open-ended problem situation, present more than one possible solution.

4. Demonstrate an ability to use problem solving techniques such as: counterexamples, use of simpler cases, looking at extremes, analysis, assuming the answer, draw a picture, working backward, guess and check, find pattern, make a table.

D. Novel problems and puzzles.

The student will:

1. Be able to solve novel problems and math puzzles.

E. Geometrical relations.

The student will:

1. When given a second or third degree equation, will draw a curve which fits the equation.

2. Use experimentation such as drawing figures, making constructions, measuring, making models, and folding paper in order to discover generalizations.

F. Conclusions from data.

The student will:

1. Identify fallacies in consumer advertising involving statistical data and graphs.

2. Detect flaws in arguments such as in advertising and propaganda.

3. Assemble and present statistical evidence in support of an argument.

G. Probability and statistics.

The student will:

1. Correctly plan a survey by defining a problem, sampling, collecting data, and reporting results.

VI. Appreciation and use of mathematics.

A. Recognizing the importance and relevance of mathematics to the individual and to society.

These are examples that should be known:

1. Role of Euclidian geometry.
2. Appreciation of Arabic numeration.
3. Recognize the names of outstanding mathematicians and know something about their work. Included should be: Gauss, Newton, Descartes, Galois, Einstein, Noether, and Pythagoras.

B. Enjoyment of mathematics.

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques, e.g., using compasses and working with numbers. Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important--e.g., the individual should not hate nor fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study and, therefore, have at his/her disposal.

C. Using the content and techniques of mathematics.

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participation in the learning of mathematics beyond that which is merely required, and actively seeking to further personal development in the area of mathematics.

The fourth subgoal related to the individual's development of a curiosity about mathematics as well as a readiness to engage in activities in this area, i.e., independent of school and/or job assignments. In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his/her skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgment or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry into adult life.

Minnesota State Department of Education